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# On Generalizing the Multiple Discrete-Continuous Extreme Value (MDCEV) Model

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# On Generalizing the Multiple Discrete-Continuous Extreme Value (MDCEV) Model

by

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## On Generalizing the Multiple Discrete-Continuous Extreme Value (MDCEV) Model

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The overall goal of the dissertation is to contribute to the growing literature on multiple discrete-continuous (MDC) choice models. In MDC choice situations, consumers often encounter two inter-related decisions at a choice instance – which alternative(s) to choose for consumption from a set of available alternatives, and the amount to consume of the chosen alternatives. In the recent literature, there is increasing attention on modeling MDC situations based on a rigorous underlying micro-economic utility maximization framework. Among these models, the multiple-discrete continuous extreme value MDCEV model (Bhat, 2005, 2008) provides a number of advantages over other models. The primary objective of this dissertation is to extend the MDCEV framework to accommodate more realistic decision-making processes from a behavioral standpoint. The dissertation has two secondary objectives. The first is to advance the current operationalization and the econometric modeling of MDC choice situations. The second is to contribute to the transportation literature by estimating MDC models that provide new insights on individuals' travel decision processes.

The proposed extensions of the MDCEV model include: (1) To formulate and estimate a latent choice set generation model within the MDCEV framework, (2) To develop a random utility-based model formulation that extends the MDCEV model to include multiple linear constraints, and (3) To extend the MDCEV model to relax the assumption of an additively separable utility function. The methodologies developed in this dissertation allow the specification and estimation of complex MDC choice models,

and may be viewed as a major advance with the potential to lead to significant breakthroughs in the way MDC choices are structured and implemented. These methodologies provide a more realistic representation of the choice process. The proposed extensions are applied to different empirical contexts within the transportation field, including participation in and travel mileage allocated to non-work activities during various time periods of the day for workers, participation in recreational activities and time allocation for workers, and household expenditures in disaggregate transportation categories. The results from these exercises clearly underline the importance of relaxing some of the assumptions made, not only in the MDCEV model, but in MDC models in general.

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# CHAPTER 1: Introduction

## 1.1 Background and motivation

Classical single-discrete choice models have been widely used to study consumer preferences for the choice of a single discrete alternative from among a set of available and mutually exclusive alternatives. However, in many choice occasions, consumers face the situation where they can choose more than one alternative at the same time, although they are not bound to choose all available alternatives. These situations have come to be labeled by the term "multiple discreteness" in the literature (see Hendel, 1999). In addition, in such situations, the consumer usually also decides on a continuous dimension (or quantity) of consumption, which has prompted the label **multiple discrete-continuous (MDC)** choice (Bhat, 2005). MDC choice situations are quite ubiquitous in consumer decision-making, and constitute a generalization of the more classical single discrete-continuous choice situation. Examples of MDC choices include:

1. Activity participation and duration: the participation decision of individuals in multiple kinds of activities within a given time (for example, over the course of a day) and the duration of the chosen activity types.
2. Vehicle type holdings and usage: households may hold a mix of different kinds of vehicle body/fuel types (for example, an electric sedan or a gas-based minivan) and use each vehicle at different rates based on several factors, such as the preferences of individual members, considerations of maintenance/running costs, and the need to satisfy different functional needs.
3. Consumer demand for products: purchase of multiple brands or types within a product category and purchased quantity. Examples include packaged food (such as soft drinks and yoghurt) and entertainment products (such as books or movies of a certain type).

There are many ways that MDC choices, such as those discussed above, may be modeled. In the recent literature, there is increasing attention on modeling these situations based on a rigorous underlying micro-economic utility maximization framework. Among these models, the **multiple-discrete continuous extreme value model MDCEV** (Bhat, 2005 and 2008) provides a number of advantages over other models: the MDCEV model has a closed-form probability expression, is practical even for situations with a large number of discrete alternatives, is the exact generalization of the multinomial logit (MNL), collapses to the MNL in the case that each decision-maker chooses only one alternative, and is equally applicable to cases with the presence or absence of an outside good. This dissertation builds upon the MDCEV model, relaxing some of the assumptions that lead to the closed probability form derived by Bhat (2005, 2008).

This chapter is structured as follows. In Section 1.2, a review of the different approaches to model MDC choices is presented. Section 1.3 presents the MDCEV model structure. In particular, the section discusses the model formulation and estimation and reviews extensions of the basic MDCEV structure. In Section 1.4, the objectives of the dissertation are defined, and Section 1.5 outlines the dissertation structure.

## 1.2 Multiple-discrete continuous (MDC) choice models

Many consumer choice situations are characterized by the simultaneous demand for multiple alternatives. Choice situations in which individuals can choose more than one alternative and a continuous dimension of the chosen alternatives are referred as multiple-discrete continuous (MDC) choices. The initial attempts toward formulating models to accommodate MDC choice structures involved the use of traditional single discrete choice models (such as the MNL). In one approach, all *bundles* of the elemental alternatives are identified, and each bundle is used as a composite alternative in a single discrete choice model. Clearly, this method is cumbersome from an empirical standpoint, especially when the number of alternatives increases. Another approach involves *stitching* together single discrete choice models. These models handle multiple discreteness through methods that generate correlation between univariate utility maximizing models for single discreteness (see Manchanda *et al*., 1999, Baltas, 2004, Edwards and Allenby, 2003, and Bhat and Srinivasan, 2005). A third approach is based on characterizing multiple discreteness as the result of a *stream* of expected future consumption decisions between successive choice occasions (see Hendel, 1999 and

Dube, 2004). These three approaches (bundle, stitching and stream) do not have a foundation in a rigorous underlying utility maximization model, though they may provide good statistical data fit.

The first MDC models based on an underlying behavioral theory can be traced to Hanemann's (1978) and Wales and Woodland's (1983) Karush-Kuhn-Tucker or KKT first-order conditions approach for constrained random utility maximization (Khun and Tucker, 1951).[1] These models adopt a direct utility approach for estimating parameters and obtaining analytic expressions for demand functions. The direct utility framework has the advantage of being closely tied to an underlying behavioral theory, so that interpretation of parameters in the context of consumer preferences is clear and straightforward. Further, it also provides insights into identification issues. This approach assumes the utility function to be random (from the analyst's perspective) over the population, and then derives the consumption vector for the random utility specification subject to the linear budget constraint by using the KKT conditions for constrained optimization.

The essential element of the utility maximization framework for multiple discreteness is the use of a non-linear (but increasing and continuously differentiable) utility structure with decreasing marginal utility (or satiation), which immediately introduces imperfect substitution and allows the choice of multiple alternatives. Further, the utility function should impose the intuitive condition of weak complementarity (see Mäler, 1974), which implies that the consumer receives no utility from a non-essential good's attributes if she/he does not consume it (see Hanemann, 1984, von Haefen, 2004, and Herriges *et al.*, 2004 for a detailed discussion on weak complementarity). Several

[1] The KKT first-order conditions provide necessary optimality conditions for nonlinear constrained problems in finite dimensional spaces. The conditions were formally introduced by Harold W. Kuhn and Albert K. Tucker in 1951, opening a new research field known as "nonlinear programming". However, the conditions were first developed in 1939 by the mathematician William Karush as part of his unpublished Master thesis at the University of Chicago. At the time of their derivation, Karush's conditions were disregarded for two reasons (Robson and Stedall, 2008). First, the optimality conditions were not the main result of his work, but only an intermediate tool. Second, Karush's thesis did not belong to the discipline of optimization theory in finite dimensional spaces, to which the conditions pertain. Karush' results were not acknowledged by the scientific community until 1975, when Khun publically recognized that Karush was the first researcher to derive the optimality conditions. Since then, the conditions initially known as Khun-Tucker (KT) became the Karush-Kuhn-Tucker (KKT) conditions.

non-linear utility specifications have been proposed in the literature, the most popular being those originating in the linear expenditure system (LES) structure or the constant elasticity of substitution (CES) structure (see Hanemann, 1978, Kim *et al.*, 2002, von Haefen and Phaneuf, 2005, and Phaneuf and Smith, 2005). Bhat (2005, 2008) proposed a utility form that is quite general and subsumes the earlier specifications as special cases. His utility specification also allows a clear interpretation of model parameters and explicitly imposes the condition of weak complementarity. In terms of stochasticity, Bhat used a multiplicative log-extreme value error term in the baseline preference for each alternative, leading to the MDCEV model, which is discussed in the following section.

### 1.3. The multiple-discrete continuous extreme value (MDCEV) model

Despite the advantages of the KKT approach discussed in Section 1.2, this approach did not receive much attention until somewhat recently because the random utility distribution assumptions used by Wales and Woodland (1983) led to a complicated likelihood function that entails multi-dimensional integration. Kim *et al.* (2002) addressed this issue by using the Geweke-Hajivassiliou-Keane (GHK) simulator (Geweke, 1991, Hajivassiliou and McFadden, 1998, and Keane, 1994) to evaluate the multivariate normal integral appearing in the likelihood function in the KKT approach. However, this approach still remained cumbersome because of the evaluation of truncated multivariate normal integrals. In contrast, the MDCEV is based on a parsimonious econometric approach to handle multiple discreteness. Indeed, the MDCEV and its variants have been used in several fields, including time-use (Chikaraishi *et al.*, 2010, Wang and Li, 2011), transportation (Rajagopalan and Srinivasan, 2008, Ahn *et al.*, 2008, Pinjari, 2011), residential energy type choice and consumption (Jeong *et al.*, 2011), land use change (Kaza *et al.*, 2009), and use of information and communication technologies (Shin *et al.*, 2009).

The MDCEV model is equally applicable to cases with complete or incomplete demand systems. In a *complete demand system*, the demands of all consumption goods are modeled. However, complete demand systems require data on prices and consumptions of all commodity/service items, and can be impractical when studying consumptions in finely defined commodity/service categories. In such situations, it is common to use an *incomplete demand system*, typically in the form of a two-stage budgeting approach or in the form of the use of a Hicksian composite commodity assumption. Whether in complete or incomplete demand systems, the elementary commodities/services in the broad group of primary interest are referred to as **inside goods**. In the Hicksian composite commodity approach, one can replace all the elementary alternatives within each broad group (that is not of primary interest) by a single composite alternative representing the broad group. The analysis proceeds then by considering the composite goods as **outside goods**. It is common in practice in this Hicksian approach to include a single outside good with the inside goods. If this composite outside good is not essential, then the consumption formulation is similar to that of a complete demand system. If this composite outside good is essential, then the formulation needs minor revision to accommodate the essential nature of the outside good. Henceforth, the case of "only inside goods" refers to the model structure for complete demand systems or the second stage of a two-stage incomplete demand system, and the case of "outside and inside goods" denotes the model structure for a Hicksian approach-based incomplete demand system.

### 1.3.1 Functional form of utility specification

#### *1.3.1.1 Case of only inside goods*

The MDCEV model is based on the general and flexible functional form for the utility function that is maximized by a consumer subject to budget constraints as follows:

$$\max U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right)$$

$$\text{s.t. } \sum_{k=1}^{K} p_k x_k = E, \qquad (1.1)$$

where the utility function $U(\boldsymbol{x})$ is quasi-concave, increasing and continuously differentiable, $\boldsymbol{x} \geq 0$ is the consumption quantity ($\boldsymbol{x}$ is a vector of dimension $K \times 1$ with elements $x_k$), and $\psi_k$, $\alpha_k$, and $\gamma_k$ are parameters associated with good $k$. The constraint in Equation (1.1) is the linear budget constraint, where $E$ is the total expenditure across all goods $k$ ($k = 1,2,...,K$) and $p_k > 0$ is the unit price of good $k$. The function $U(\boldsymbol{x})$ is a valid utility function if $\psi_k > 0$, $\gamma_k > 0$, and $\alpha_k \leq 1$ for all *k*. The utility function form clarifies the role of each of the $\psi_k$, $\alpha_k$, and $\gamma_k$ parameters. In particular, $\psi_k$ represents the baseline marginal utility, or the marginal utility at the point of zero consumption. $\gamma_k$ is the vehicle to introduce corner solutions for good *k* (that is, zero consumption for good *k*), but also serves the role of a satiation parameter. Finally, the express role of $\alpha_k$ is to capture satiation effects. To understand the role of $\gamma_k$ and $\alpha_k$ on consumer preferences, Figures 1.1 and 1.2 show the effect of the parameters $\gamma_k$ and $\alpha_k$, respectively, on the marginal utility function:

- Figure 1.1 plots the marginal utility function of good $x_k$ for different values of $\gamma_k$, and $\alpha_k = 0.1$ and $\psi_k = 1$. All the curves have the same value ($\psi_k = 1$) when $x_k = 0$, and then decrease with $x_k$ at different rates. The figure shows that lower satiation effects (or stronger preferences) are reached for higher values of $\gamma_k$.
- Figure 1.2 plots the marginal utility function of good $x_k$ for different values of $\alpha_k$, and $\gamma_k = 1$ and $\psi_k = 1$. When $\alpha_k \to 1$, this represents the case of absence of satiation effects or, equivalently, the case of constant marginal utility (that is, the case of single discrete choice). As $\alpha_k$ moves downward from the value of 1, the satiation effect for good *k* increases.

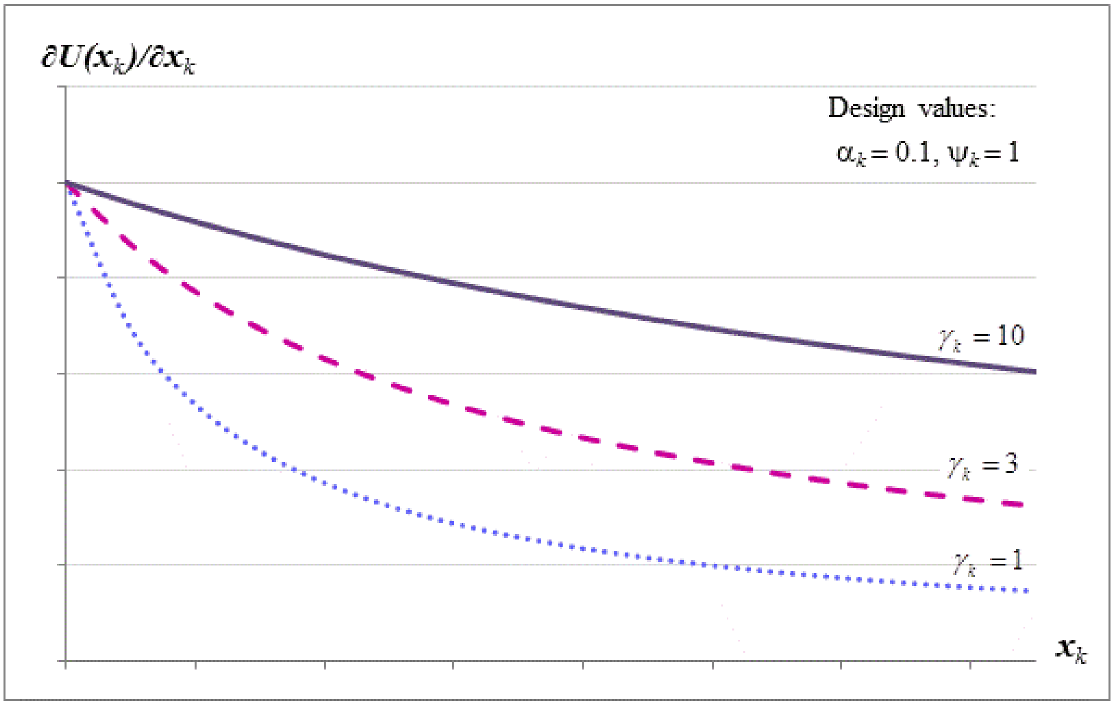


Figure 1.1: Effects of $\gamma_k$ on the marginal utility function

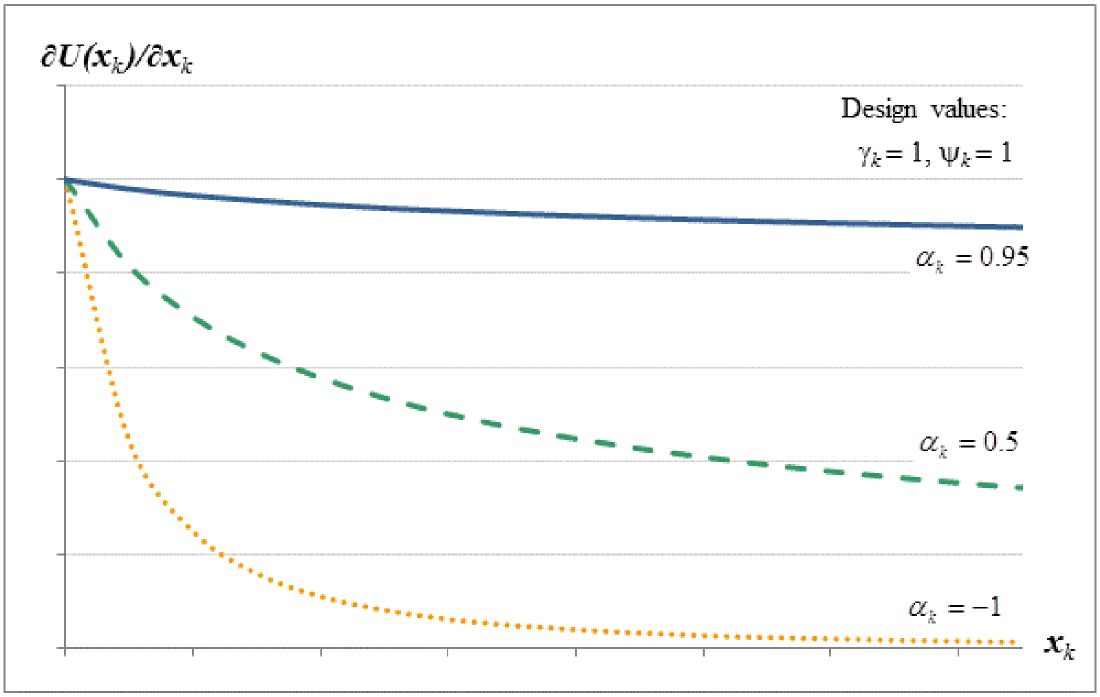


Figure 1.2: Effects of $\alpha_k$ on the marginal utility function

Empirically speaking, it is difficult to disentangle the two effects of the $\gamma_k$ and $\alpha_k$ parameters separately, which leads to serious empirical identification problems and estimation breakdowns when one attempts to estimate both $\gamma_k$ and $\alpha_k$ parameters for each good. Thus earlier studies have either constrained $\alpha_k$ to zero for all goods (technically, assumed $\alpha_k \to 0 \ \forall k$) and estimated the $\gamma_k$ parameters, or constrained $\gamma_k$ to 1 for all goods and estimated the $\alpha_k$ parameters. The first case is referred as the $\boldsymbol{\gamma}$ **profile**, and the second case is known as the $\boldsymbol{\alpha}$ **profile** (also referred as the linear expenditure system (LES) form), as presented in Equation (1.2). This issue is discussed in detail by Bhat (2008), who suggests testing both these normalizations and selecting the model with the best fit.

$$
\begin{aligned}
&\gamma \text{ profile} \qquad U(\boldsymbol{x}) = \sum_{k=1}^{K} \gamma_k \psi_k \ln\left(\frac{x_k}{\gamma_k} + 1\right) \\
&\alpha \text{ profile} \qquad U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\psi_k}{\alpha_k}\left((x_k + 1)^{\alpha_k} - 1\right).
\end{aligned}
\tag{1.2}
$$

To find the optimal allocation of goods, we construct the Lagrangian and derive the Karush-Kuhn-Tucker (KKT) conditions. The Lagrangian function for the model of Equation (1.1) is:

$$
L = U(\boldsymbol{x}) + \lambda\left(E - \sum_{k=1}^{K} p_k x_k\right), \tag{1.3}
$$

where $\lambda$ is the Lagrangian multiplier for the budget constraint, which represents the marginal utility of expenditure. The KKT first order conditions for optimal consumption allocations ($x_k^*$) are:

$$
\begin{aligned}
&\psi_k\left(\frac{x_k^*}{\gamma_k} + 1\right)^{\alpha_k - 1} - \lambda p_k = 0 \quad \text{if } x_k^* > 0,\ k = 1,2,\ldots,K \\
&\psi_k\left(\frac{x_k^*}{\gamma_k} + 1\right)^{\alpha_k - 1} - \lambda p_k < 0 \quad \text{if } x_k^* = 0,\ k = 1,2,\ldots,K\,.
\end{aligned}
\tag{1.4}
$$

The optimal demand satisfies the conditions in Equation (1.4) plus the budget constraint. The budget constraint implies that only $K-1$ of the optimal consumptions need to be estimated, since the quantity consumed of any one good is automatically determined from the quantity consumed of all the other goods. To accommodate this constraint, designate good 1 as a good to which the individual allocates some non-zero amount of consumption (note that the individual should participate in at least one of the $K$ purposes, given that $E>0$). For the first good, the KKT condition may then be written as:

$$\lambda = \frac{\psi_1}{p_1}\left(\frac{x_1^*}{\gamma_1}+1\right)^{\alpha_1-1}. \tag{1.5}$$

Substituting $\lambda$ for from above into Equation (1.4) for the other goods ($k=2,...,K$), and taking logarithms, we can rewrite the KKT conditions as:

$$\begin{aligned} &v_k + \ln\psi_k = v_1 + \ln\psi_1 \quad \text{if } x_k^* > 0,\ k=2,...,K \\ &v_k + \ln\psi_k < v_1 + \ln\psi_1 \quad \text{if } x_k^* = 0,\ k=2,...,K, \end{aligned} \tag{1.6}$$

where $v_k = (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k}+1\right) - \ln p_k$, $(k=1,2,...,K)$.

*1.3.1.2 Case of outside and inside goods*

If outside goods and inside goods are present, label the outside goods as the first $K_1$ bundle of goods which now has a unit price of one, and label the inside goods as the following $K_2$ bundle of goods $(K_1+K_2=K)$. Then, the utility functional form of Equation (1.1) needs to be modified as follows:

$$U(\boldsymbol{x}) = \sum_{j=1}^{K_1}\frac{\psi_j}{\alpha_j}\left(x_j+\gamma_j\right)^{\alpha_j} + \sum_{k=K_1+1}^{K}\frac{\gamma_k}{\alpha_k}\psi_k\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k}-1\right). \tag{1.7}$$

In the above formula, we need $\gamma_j > 0$ and $x_j+\gamma_j > 0$, $j=1,2,...,K_1$. The magnitude of $\gamma_j$ may be interpreted as the required lower bound (or a “subsistence value”) for consumption of the outside goods. As in the only-inside-goods case, the analyst will generally not be able to estimate both $\gamma$ and $\alpha$ for the outside and inside goods, and should choose between the $\gamma$ profile and $\alpha$ profile.

The same procedure undertaken for the only-inside-goods case can be derived, and the KKT conditions are the same as in Equation (1.6), replacing $v_j = (\alpha_k - 1)\ln(x_k^* + \gamma_k) - \ln p_k$ for the outside goods ( $j = 1,2,...,K_1$ ).

1.3.2 Model estimation

The baseline random marginal utility for each good is defined as:

$$\psi_k = \exp\left(\boldsymbol{\beta}'\boldsymbol{z}_k + \varepsilon_k\right), \quad k = 1,2,...,K \,, \tag{1.8}$$

where $\boldsymbol{z}_k$ is a set of attributes that characterize alternative *k* and the decision maker (including a constant), and $\varepsilon_k$ captures the idiosyncratic (unobserved) characteristics that impact the baseline utility of good *k*. This parameterization guarantees the positivity of the baseline utility. Substituting $\psi_k$ in the KKT conditions of Equation (1.6), we obtain:

$$\begin{aligned} V_k + \varepsilon_k &= V_1 + \varepsilon_1 \quad \text{if } x_k^* > 0\,,\ k = 2,...,K \\ V_k + \varepsilon_k &< V_1 + \varepsilon_1 \quad \text{if } x_k^* = 0\,,\ k = 2,...,K\,, \end{aligned} \tag{1.9}$$

where $V_k = v_k + \boldsymbol{\beta}'\boldsymbol{z}_k$, $k = 1,2,...,K$ .

To complete the model structure, the analyst needs to specify the error structure. Assuming that the error terms $\varepsilon_k$ are independently distributed across all alternatives $(k = 1,2,...,K)$ and independent of $\boldsymbol{z}_k$, and follow a standard extreme value distribution with scale parameter $\sigma$, the probability that the individual allocates expenditure to the first *M* of the *K* goods collapses to the following closed expression (see Bhat, 2008 for details):

$$P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = det(J)\frac{1}{\sigma^{M-1}}\left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right](M-1)!, \tag{1.10}$$

where the determinant of the Jacobian $J$ has a closed form:

$$det(J) = \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{p_i}{c_i}\right], \text{ where } c_i = \frac{1-\alpha_i}{x_i^* + \gamma_i}. \quad (1.11)$$

In the case when $M = 1$ (*i.e.*, only one alternative is chosen), there are no satiation effects and the Jacobian term drops out (that is, the continuous component drops out, because all expenditure is allocated to good 1). In this case, the model in Equation (1.10) collapses to the standard MNL model.

### 1.3.3 Extensions of the MDCEV model

Most extensions of the MDCEV model have been focused on relaxing the assumptions made over the distribution of the error term.[2] In the original MDCEV study, Bhat (2008) provides the general econometric model structure that can be used with different joint probability density functions. In particular, Bhat proposes the MDC generalized extreme-value (GEV) model, that allows correlation across alternatives using a GEV error structure, and the mixed MDCEV model, focusing on heteroscedastic structures. The MDC-GEV was later studied in detail by Pinjari (2011), while the mixed MDCEV model has received significant attention in recent years in the transportation field (see Bhat, 2005, Sener and Bhat, 2006, Kapur and Bhat, 2007, Shin *et al.*, 2012).

Other extensions of the MDCEV include: Pinjari *et al.* (2009), who account for self-selection effects of residential location in activity time use participation via a joint mixed multinomial logit (MLN) - mixed MDCEV framework; Pinjari and Bhat (2008), who study a MDC nested extreme value model applied to non-worker activity time-use; Spissu *et al.* (2009), who formulated a panel version of the mixed MDCEV model and applied it to time use models; and Khan *et al.* (2012), who employed the MDCEV utility function but, instead of using an extreme value error term, introduced a multivariate normal error with a normal mixing structure. The above extensions have been focused on incorporating more general stochastic specifications that can allow for general patterns of

[2] An exception is Vásquez-Lavín and Hanemann (2008) study, in which a non-additively separable MDCEV model structure is formulated to study angler site choice. This model is reviewed in detail in Chapter 4.

correlation, while retaining, at some extent, the simple expressions of the MDCEV. Although these models can potentially provide econometric flexibility, little research has been dedicated to extend the MDCEV to capture more realistic behavioral representations of the consumer choice process.

### 1.4 Current research

Econometric models of MDC choice have been extensively used in the transportation field to develop and estimate travel demand models. Although there are plenty of examples of research studies in the transportation and economics literature that document the importance of incorporating behavioral realism in the choice process, there has been a relatively small body of literature explicitly accommodating this awareness into the development of appropriate mathematical modeling techniques in the context of MDC choice modeling.

The objective of this dissertation is to extend the MDCEV framework to accommodate more realistic decision-making processes from a behavioral standpoint. The proposed extensions of the MDCEV model include the following.

1. To formulate and estimate a latent choice set generation model within the MDCEV framework. This choice set generation model can be used to determine the set of alternatives that each individual will consider, while recognizing the fact that the consideration choice set is not explicitly observed by the analyst. This extension also allows flexibility to accommodate non-compensatory behavior in the choice process through the incorporation of varying choice sets across individuals.
2. To develop a random utility-based model formulation that extends the MDCEV model to include multiple linear constraints. The formulation uses a flexible and general utility function form that is applicable to the case of complete demand systems as well as incomplete demand systems. The proposed research allows for the presence of any number of outside goods and shows how the Jacobian structure has a closed-form structure for most MDC situations, which aids in estimation.

3. To extend the MDCEV model to relax the assumption of an additively separable utility function. The proposed utility functional form remains within the class of flexible forms, while also retaining global theoretical consistency properties. The form also allows clarity in the interpretation of parameters and helps understand identification issues. In addition, with specific ways of introducing stochasticity, the formulation retains a relatively simple form for the model. The structure of the Jacobian in the likelihood function is also relatively simple because of the way stochasticity is introduced.

The methodologies developed in this research allow the specification and estimation of complex MDC choice models, and may be viewed as a major advance with the potential to lead to significant breakthroughs in the way MDC choices are structured and implemented. The proposed extensions are applied to different empirical contexts within the transportation field, including participation in and travel mileage allocated to non-work activities during various time periods of the day for workers, participation in recreational activities and time allocation for workers, and household transportation expenditures in disaggregate categories.

## 1.5 Proposal outline

The rest of the dissertation proposal is structured as follows. Chapter 2 presents the first extension of the MDCEV model related to the inclusion of a first decision stage on the considered alternatives, based on a latent choice set generation approach. Chapter 3 describes the modeling framework to accommodate multiple linear resource constraints within the MDCEV model. Chapter 4 provides a methodological framework to relax the additively-separable assumption on the utility form assumed in the MDCEV model. The last and the final chapter concludes the dissertation by summarizing the findings in the previous chapters, discussing some limitations of the current work, and suggesting directions for future research.

# CHAPTER 2: Accommodating a Latent Choice Set Generation in the MDCEV Model

> The material in this chapter is draws substantially from the following published paper:
> Castro, M., N. Eluru, C.R. Bhat and R.M. Pendyala (2011) Joint model of participation in nonwork activities and time-of-day choice set formation for workers. *Transportation Research Record* 2254, 140-150.

This chapter provides the methodological details to implement a latent choice set generation within the MDCEV framework. Section 2.1 discusses the behavioral paradigm and modeling considerations that shaped the structure and specification of the model system. Section 2.2 presents the modeling methodology and formulation of a two-stage approach involving choice set generation in the context of a MDC choice situation. Section 2.3 describes an empirical application of the proposed framework to jointly model the participation decision in non-work activities and the corresponding travel mileage of workers.

## 2.1 Choice set generation

In the context of the behavioral choices, a choice set is the set of alternatives that are relevant to the individual's choices. It is entirely possible that some individuals may not consider all the available alternatives when making their choices. Instead, certain individuals – depending on a variety of factors – may consider only a subset of alternatives. In other words, researchers must recognize the possibility that the choice set is not constant, but variable, across the population. This requires the inclusion of a component capable of modeling choice set generation or composition within the framework adopted for the choice model.

The importance of choice set consideration has been recognized widely in the transportation literature (see, for example, Williams and Ortúzar, 1982, Swait, 2001, Basar and Bhat, 2004, Kaplan and Prato, 2012). Unfortunately, as in many choice contexts, it is not possible to explicitly identify the choice set for each individual as such information is virtually never included in an activity-travel survey data set or in other choice preference surveys. Then, the analyst must determine the feasible choice set for each individual based on a variety of criteria or rules. Manski (1977) proposed a two-stage approach (or two-step model) for tackling problems of this nature. In the first stage, the choice set is generated as a subset of the universal choice set, and in the second stage, the individual selects alternatives conditional on the choice set. Some applications of this approach in the single discrete choice context can be found in Basar and Bhat (2004), McFadden (1978), Swait and Ben-Akiva (1987) and Cantillo and Ortúzar (2005).

Another important reason for modeling choice set consideration is the flexibility to accommodate non-compensatory behavior in the choice process. If a choice alternative does not meet the constraints or conditions for its inclusion, then it is eliminated from the choice set regardless of its attributes and its relation to other choice alternatives in the choice set. Estimating a compensatory model ignoring such non-compensatory behavior will lead to incorrect estimation of the impacts of variables on choice dimensions of interest. A latent choice set generation model that recognizes the latent (unobserved or hidden) nature of the choice set determination process allows accommodating compensatory choice behavior.

There has been considerable work on the development of latent choice set generation models. Swait and Ben-Akiva (1987) indicate that choice set formation is a constrained process that should consider informational, psychological, cultural, and social restrictions. Shocker *et al.* (1991) identify four levels of choice set formation, including the universal set of all alternatives, the awareness set, the consideration set, and finally, the actual choice set. Ben-Akiva and Boccara (1995) develop a probabilistic choice set generation model considering individual heterogeneity with a focus on incorporating the effects of non-compensatory mechanisms of choice and influence of attitudes and perceptions on the choice process. Swait (2001) proposed a choice set generation model that belongs to the generalized extreme value (GEV) class of models. Cantillo and Ortúzar (2005) employ attribute thresholds to eliminate alternatives from the choice set, with the attribute thresholds varying across individuals based on socio-economic and demographic characteristics.

The uniqueness of the current study is that the two-stage approach involving choice set generation is employed in the context of a MDC choice situation. An exception is the work by von Haefen (2008) who does employ a two-stage approach in the context of a MDC choice problem, but his model formulation is different from that of the MDCEV (Bhat, 2008) which offers a more computationally tractable closed form expression for parameter estimation.

## 2.2 Methodology

The model structure used in the research effort is based on Manski's (1977) original two-stage choice paradigm. The adopted structure includes a probabilistic choice set generation model in the first stage, followed by modeling discrete-continuous choice dimensions in the MDC context given the choice set from the first stage.

The first stage uses a probabilistic choice set generation mechanism because the actual choice set of alternatives is unobserved to the analyst and, therefore, cannot be determined with certainty by the analyst. Within the class of probabilistic choice set generation models, this study adopts the Swait and Ben-Akiva (1987) random constraint-based approach to choice set formation. In the random constraint-based approach, an alternative is included in the choice set if the consideration utility for that alternative is greater than some threshold consideration utility level. The consideration utility is allowed to vary across individuals, so that the consideration probability of each alternative varies across individuals.

The second stage model, given the choice set, is based on the MDCEV approach. At this stage, the traditional random utility maximizing process is at play wherein utilities of the alternatives in the choice set are compared directly with each other. The difference in the process at the choice set generation and choice determination stages enables a change in an attribute associated with an alternative to have two separate effects: a consideration effect (*i.e.*, the impact on the consideration set of alternatives) and a choice effect (*i.e.*, the impact on the choice of an alternative, given that the alternative is considered by the individual).

The model formulation in this section is developed assuming that all alternatives are feasible for each individual. An alternative $k$ ($k = 1,2,...,K$) is included in the choice set if this consideration utility exceeds a certain threshold and is eliminated otherwise. As the threshold is not observed by the analyst, it is considered a random variable. This random threshold is assumed to be standard logistically distributed. Then, the probability that alternative $k$ is considered by individual $q$ can be written as:

$$S_{qk} = \frac{1}{1+\exp(-\boldsymbol{\delta}'\boldsymbol{w}_{qk})}, \tag{2.1}$$

where $\boldsymbol{w}_{qk}$ is a column vector of observed attributes for individual $q$ and alternative $k$ (including a constant) and $\boldsymbol{\delta}$ is a corresponding column vector of coefficients to be estimated (this set of coefficients provides the impact of characteristics on the consideration probability of alternative $k$). Given the previous expression, the threshold ($\boldsymbol{\delta}'\boldsymbol{w}_{qk}$) is a function of individual, socio-demographic and environmental characteristics. Next, assume that the randomly-distributed threshold for each alternative is independent of the threshold values of other alternatives. The overall probability of a choice set $C$ for individual $q$ may then be written as:

$$P_q(C) = \frac{\prod_{k \in C} S_{qk} \prod_{k \notin C} (1 - S_{qk})}{1 - \prod_{k} (1 - S_{qk})}, \tag{2.2}$$

where the denominator is a normalization to remove the choice set with no alternatives in it. In the second stage of the MDCEV model, the choice of an alternative or set of multiple alternatives from a given choice set can be written as (see Section 1.3.2):

$$P_q \mid C = \frac{1}{\sigma^{M-1}}\left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M}\frac{p_i}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^{M}}\right](M-1)! \qquad (2.3)$$

Finally, the unconditional probability of choice of alternative *q* may be derived as:

$$P_q = \sum_{C\in G}\left(P_q \mid C\right)P_q(C) \qquad (2.4)$$

where *G* is the set of all nonempty subsets of the master choice set of all alternatives. The membership of *G* could include ($2^I - 1$) elements. For example, in a three alternative case, denoted as {A, B, C}, *G* includes the following choice sets: {A}, {B}, {C}, {A, B}, {B, C}, {A, C} and {A, B, C}.

## 2.3 Application to non-work activities and travel participation

### 2.3.1 Empirical context

Urban areas around the world are experiencing increasing levels of travel demand and vehicular miles of travel (VMT), particularly in rapidly growing regions of the globe (Pendyala and Kitamura, 2007). Although transportation professionals have traditionally focused on work-related travel and the commute journey in an effort to manage peak period congestion, it is becoming increasingly clear that non-work travel demand, which tends to be more discretionary and exhibits greater variability across the population, is a critically important component of overall travel demand in metropolitan regions. Evidence in the literature suggests that workers are increasingly participating in non-work activities, particularly in conjunction with the commute to or from work. Gordon *et al.* (1988) measured the growth of non-work travel using the 1977 and 1983 Nationwide Personal Transportation Survey (NPTS) in the U.S., and particularly noted the growth in such travel during the work-to-home commute. Lockwood and Demetsky (1994) also noted that a large number of individuals made one or more stops during the return home commute journey. Strathman and Dueker (1995) analyzed the 1990 NPTS data and noted that nearly 20% of non-work activities were part of the daily commute for workers. More recently Hu and Young (1999) and Toole-Holt *et al.* (2005) reported that increases in overall travel demand may be largely attributed to growth in non-work travel. McGuckin *et al.* (2005) report an increase in trip chaining, particularly among men on the journey from home to work, and note that this increase in trip chaining is largely due to non-work stops for coffee and breakfast.

The need to accurately model non-work activity participation and associated travel is also critical in the context of the development and specification of activity-based travel model systems that focus on tours (or trip chains) as the unit of analysis. In these model systems, travel patterns are simulated for each individual in a synthetic population while recognizing that individual trips do not exist in isolation, but are often linked or chained together into tours. In a tour-based framework, one is interested in modeling non-work activity stops that may occur in different tours, and the travel associated with such stops.

Given the importance of, and increasing emphasis being placed on, modeling non-work travel engagement, we provide a framework for jointly modeling worker's participation in and miles of travel for non-work travel in time-of-day blocks or periods that can be defined in relation to the work schedule. For workers, it is possible to identify five time periods for non-work activities, as depicted in Figure 2.1 (Bhat and Singh, 2000, Rajagopalan *et al.*, 2009):

- Before work tour: activities that are part of tours that start and end at home prior to the commencement of the first work episode of the day.
- During home-to-work tour: non-work activities undertaken on the way to work
- Work based tour: non-work activities undertaken as part of tours that begin and end at the work location
- During work-to-home tour: non-work activities undertaken on the way home from work
- After work tour: non-work activities undertaken as part of separate home-based tours made after arriving home from work

There are several key dimensions worth noting in the context of the behavioral choices considered. First, there is a continuous choice element represented by the amount of mileage devoted to non-work travel. Second, and more consequential to the contribution of this dissertation, is the multiple discrete nature of the choice of whether to participate in non-work activities during the defined time periods. Individuals may choose to participate in non-work activities during none, one, or more than one period identified previously. Thus, the choice of period in which to participate in a non-work activity is not a single discrete choice problem, but a multiple discrete problem. The total mileage in non-work activity-related travel is apportioned or allocated across the non-work activity engagement in the various time-of-day blocks. This leads one to adopt the MDCEV model, which offers an appropriate approach for jointly modeling non-work activity participation during a time period as well as the mileage traveled to pursue such activities, effectively tying activity engagement with the associated travel mileage. Further, in the context of the behavioral choices modeled in application, it is entirely possible that some individuals may not consider all five time-of-day blocks for undertaking non-work activities. To include this phenomenon, a latent choice set generation model is included as a first component in the model system, following the framework proposed in Section 2.2.

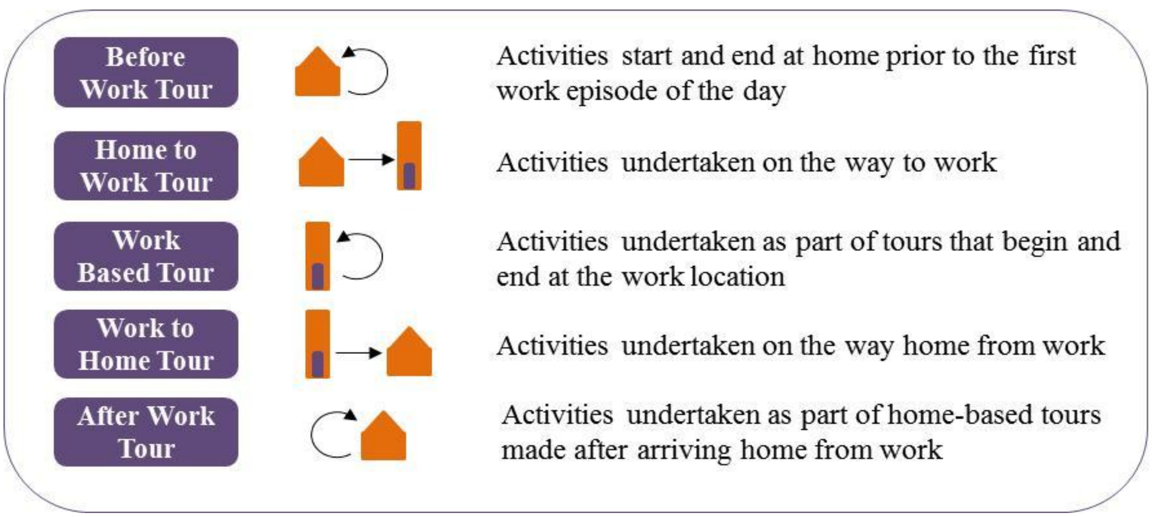


Figure 2.1. Time-of-day block periods

2.3.2 Data description

The data used in this study is derived from the 2009 National Household Travel Survey (NHTS) conducted in the U.S. The 2009 NHTS collected detailed information on more than one million trips undertaken by 320,000 individuals from 150,000 households sampled from all over the country. The survey also collected detailed information on individual and household socio-demographic and employment-related characteristics.

The focus of this application is on modeling non-work travel behavior for employed individuals. In order to have a manageable data sample for the modeling effort of this study, only the survey subsample corresponding to employed individuals residing in the San Francisco Bay Area of California was used. San Francisco was chosen for two main reasons. First, San Francisco has a substantially higher share of transit ridership compared to many parts of the country. Second, we have prepared detailed built environment data for the San Francisco urban region. While the 2009 NHTS data set does not, at this time, include geocoded residential and activity location information, it is anticipated that this information will become available in some form in the near future. When that happens, the built environment data we have for the San Francisco area can be used for further research.

The process of generating the estimation sample for analysis involved several steps. First, only employed individuals aged 18 years and above that participated in at least one work activity episode were selected. Second, only those who provided travel information for a weekday were included in the estimation sample. Third, a small sample of individuals that traveled for unusually long distances was excluded from the estimation sample (less than one percent). Fourth, records that contained incomplete information on individual, household, activity and travel characteristics were removed from the sample. Finally, several consistency checks were performed and records with missing or inconsistent data were eliminated. After the data cleaning process, the final estimation sample contained observations on 1128 individuals.

The non-work activity participation information was organized for each individual in a manner consistent with the framework proposed in Section 2.3.1. Tours were defined as per the categories identified earlier, with each tour corresponding to a period in relation to the work episode(s). Once non-work activities were placed into the appropriate tours and time-of-day periods, the vehicle miles traveled for non-work activities is aggregated within each period to obtain the continuous mileage variable. These measures were computed separately for auto, transit, and non-motorized modes of transportation resulting in a total of 15 possible alternatives for each individual (five time periods crossed with three modes of transportation). However, as the transit and non-motorized mode samples were very small, the ten alternatives associated with these modes of transport had to be collapsed into a single non-auto alternative. With this consolidation of alternatives, the final number of alternatives in the universal choice set is six – five auto alternatives and one non-auto alternative.

The mileage computation considers only the distance traveled for non-work travel (activities). In the database, trip distance was self-reported. Hence, it is possible for some individuals to have zero mileage for all six alternatives. Indeed, it is found that 31.4% of the sample does not pursue any non-work activities in an entire day. To facilitate the estimation of the MDCEV model, a very small mileage (0.2 miles) is added to the non-auto mode alternative for individuals who did not report any intermediate stops. This small manipulation is primarily undertaken to ensure that the MDCEV model can be estimated, and has no impact on model estimation results.

Table 2.1 shows sample statistics for non-work activity participation by time period and associated travel mileage. Mileage statistics are reported for the subsample that actually participated in a non-work activity within any specified time period. It should be noted that the sample shares of the chosen alternatives do not add up to 100% because individuals can participate in more than one activity, and in more than one time period, in a day. The descriptive statistics suggest that there is a higher inclination to undertake non-work activities after work (during work to home and after work tours) than before work. As described in earlier literature (Strathman and Dueker, 1995, Bhat and Sardesai, 2006), trip chaining on the way from work to home is more prevalent than trip chaining on the way to work from home. As expected, the average mileage for the non-auto alternative is lower than for auto alternatives. Also, non-work related travel mileage is lower in the context of home-to-work and work-to-home tours; this finding is consistent with the notion that non-work stops made on the way to or from work are likely to constitute minor deviations from the home-work path. It should be noted that only the additional mileage that can be clearly attributed to the non-work activity or stop is included in the computations of mileage in this table.

Table 2.1: Non-work activity participation and mileage statistics

| **Time-of-Day Period** | **Percent Participating** | **Average Mileage** | **Std. Dev. of Mileage** | **Mileage Percentiles** | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | **10th** | **25th** | **50th** | **75th** | **90th** |
| Before Work Tour | 6.6 | 12.64 | 13.76 | 1.90 | 2.24 | 8.05 | 15.34 | 31.69 |
| Home to Work Tour | 17.5 | 6.13 | 12.64 | 0.18 | 0.78 | 2.14 | 5.67 | 12.17 |
| Work Break Tour | 11.6 | 12.13 | 15.38 | 2.01 | 3.02 | 7.04 | 15.09 | 25.15 |
| Work to Home Tour | 28.3 | 7.11 | 11.70 | 0.13 | 1.07 | 3.23 | 8.13 | 16.85 |
| After Work Tour | 19.1 | 12.14 | 12.23 | 2.01 | 4.02 | 8.05 | 14.08 | 31.43 |
| Non-Auto Tour | 20.6 | 3.44 | 5.41 | 0.20 | 0.20 | 0.20 | 1.11 | 4.00 |

Table 2.2 provides a summary of select household and personal characteristics for the final sample used for estimation. The analysis reveals a slightly higher proportion of females and individuals aged over 50 years in the sample. At the household level, about 80% of the households have either one or two adults, with nearly 56% of the households reporting having no children. The focus on employed individuals and the San Francisco Bay area appears to result in a substantial proportion of high income (>$100,000 per year) households.

Table 2.2: Independent variables characteristics

| Characteristics | Sample Share [%] |
|---|---|
| **Individual Level** | |
| *Age (years)* | |
| 18 to 30 | 8.1 |
| 31 to 40 | 18.3 |
| 41 to 50 | 29.9 |
| Greater than 50 | 43.8 |
| *Gender* | |
| Female | 53.9 |
| Male | 46.1 |
| **Household Level** | |
| *Number of Adults* | |
| 1 | 12.1 |
| 2 | 67.5 |
| 3 | 13.9 |
| 4 or more | 5.5 |
| *Presence of Children* | |
| No children | 55.9 |
| 0 to 5 years | 13.0 |
| 6 to 15 years | 22.4 |
| 15 to 18 years | 8.6 |
| *Household Income (US dollars)* | |
| < 35,0000 | 6.2 |
| 35,000 to 100,000 | 36.9 |
| >100,000 | 56.9 |

### 2.3.3 Model estimation results

This section presents detailed description of the model estimation results. A variety of explanatory variables were considered in the model specification including individual socio-demographics, household socio-demographics, work-related characteristics, mobility and situational characteristics, and household location characteristics. A number of alternative model forms were explored for the MDCEV component of the model. Because the non-auto alternative was always chosen, the utility function of the MDCEV model was modified to include one outside good (Equation (1.7)). In this study, it was found that the data fit was superior for the $\gamma$ profile (see Section 1.3.1, Equation (1.2)). The final model specification was obtained in a systematic manner by adding variables sequentially and examining the coefficients with respect to statistical significance, sign and magnitude, and intuitive behavioral plausibility. The selection of variables was also driven by insight from earlier empirical work on non-work activity participation and travel mileage modeling. Various forms of the explanatory variables, including non-linear, spline, and interaction effects, were considered and tested. The final specification obtained after this judicious procedure is presented in Tables 2.3 and 2.4.

Before discussing the detailed model estimation results, it is useful to review the goodness-of-fit statistics of alternative model forms to assess whether the latent MDCEV model structure offers a superior data fit in comparison to a model that does not account for latent choice set generation processes. For this effort, three models were estimated. First, the latent MDCEV model proposed in this study was estimated to give due consideration to the choice set generation process unobserved by the analyst. Second, a pure MDCEV was estimated as a restriction of the latent MDCEV using the same choice specification as the latent MDCEV but without the choice set generation component (the results of the pure MDCEV are in Appendix A). The comparison of these model results highlights three primary differences in variable effects. First, a large number of variables are statistically insignificant in the pure MDCEV model (including the Asian male, age between 18 and 30 years, female with very young children, flexible work start time, and household location variables). Second, the effects of variables on the choice process differ substantially across the two models. Finally, some of the variables that were included in the latent MDCEV model in both stages have inflated estimates in the pure MDCEV model, reinforcing the idea that the pure MDCEV co-mingles effects of variables on choice set formation and the choice decision.

From a data fit standpoint, the log-likelihood measure for the latent MDCEV model is -3,129.6 with 74 parameters. The corresponding figure for the pure MDCEV model is much lower at -6,617.6 with 69 parameters. Although the improvement in log-likelihood measures is readily apparent, it is useful to undertake a more rigorous statistical test to compare the model fits. The two models are not directly nested within one another thus eliminating the possibility of using the likelihood ratio test for comparing model specifications. Therefore, the adjusted $\rho^2$ test statistic and the Bayesian Information Criterion (BIC) measure are used.

The adjusted likelihood ratio index compares the fit of the estimated model with respect to the log-likelihood at the market shares according to the following equation:

$$\rho^2 = 1 - \frac{L(\hat{\boldsymbol{\beta}}) - N_{par}}{L(c)}, \tag{2.5}$$

where $L(\hat{\boldsymbol{\beta}})$ and $L(c)$ are the log-likelihood function at convergence and market shares respectively, and $N_{par}$ is the number of estimated parameters (excluding the constants of the choice model). The adjusted likelihood ratio index for the latent MDCEV model is 0.608, while that for the pure MDCEV model is 0.182. The results indicate that the latent MDCEV is substantially preferred over the MDCEV.

The Bayesian Information Criterion (BIC) is given by the expression:

$$BIC = -2L(\hat{\boldsymbol{\beta}}) + N_{par} \cdot \ln(Q), \tag{2.6}$$

where $L(\hat{\boldsymbol{\beta}})$ is the log-likelihood function at convergence, $N_{par}$ is the number of parameters, and $Q$ is the sample size. The model with the lower BIC value is the preferred one. The BIC value for the latent MDCEV model is 6,779.3, which is substantially lower than that for the MDCEV model which has a BIC of 13,720.1. These results clearly illustrate the superior data fit offered by the latent MDCEV model.

Table 2.3: Latent MDCEV results - Latent choice set generation component *(figures in parentheses are t-statistics)*

| Explanatory Variables | Before Work | Home to Work | Work Based | Work to Home | After Work | Non-Auto |
|---|---|---|---|---|---|---|
| *Individual demographics* | | | | | | |
| Age | -0.0351 (-2.564) | | | | | |
| Female | | | | 2.3050 (3.182) | 1.0234 (2.690) | |
| Asian | | | -1.3339 (-1.913) | | | |
| Asian Male | | 0.9101 (1.588) | | | | |
| Hispanic Male | | -1.3736 (-1.922) | | -1.2826 (-1.834) | | |
| Without a driver license | | | | | | 1.233 (1.491) |
| *Household socio-demographics* | | | | | | |
| Number of persons | | | | | 0.4398 (2.383) | |
| Presence of very young children | | | | | | -0.6746 (-3.604) |
| Presence of young children | | | 1.3786 (1.891) | | | |
| Vehicle Availability | 0.6814 (2.424) | | | | | |
| Low annual household income | | | | | | 0.4667 (1.648) |
| Medium annual household income | -1.101 (-2.819) | | | | | |
| Housing unit is owned | | | | 0.8387 (1.725) | | |
| *Work-related characteristics* | | | | | | |
| Flexible start time | | | | 1.1271 (2.538) | -0.5454 (-1.609) | |
| Have more than one job | | 1.3739 (1.616) | | | | |
| Self-employed | 0.8217 (1.856) | 1.3347 (2.524) | 1.4903 (2.385) | | | |
| Distance to work < 2 miles | 1.4025 (2.892) | | | | | |
| *Mobility and situational characteristics* | | | | | | |
| Number of bike trips in past week | | | | | | 0.0982 (2.028) |
| Number of walk trips in past week | | | | | | 0.0482 (3.788) |
| Trip was made alone | -1.8695 (-4.787) | -2.2159 (-4.764) | -1.8875 (-2.916) | | -2.6404 (-5.242) | |
| Monday | 0.9922 (2.591) | | | | -1.1145 (-2.381) | |
| Friday | | | -1.5813 (-2.008) | | | |
| *Household location variables* | | | | | | |
| Not in urban area | | | | | -1.8415 (-2.041) | |
| Urban size < 1 million | | | -1.2404 (-2.460) | | | |
| Urban size > 1 million with access to subway or rail | | | -1.4221 (-3.191) | -1.2268 (-2.630) | | |
| *Constant* | -0.4278 (-0.549) | 0.9377 (1.953) | 1.8486 (2.282) | -0.2550 (-0.493) | 0.2235 (0.334) | - |

Table 2.4: Latent MDCEV results - MDCEV component (*figures in parentheses are t-statistics*)

| **Explanatory Variables** | **Before Work** | **Home to Work** | **Work Based** | **Work to Home** | **After Work** | **Non-Auto** |
|---|---|---|---|---|---|---|
| *Individual demographics* | | | | | | |
| Age 18 to 30 years | | | -1.3679 (-1.460) | | -1.2539 (-2.908) | |
| *Household socio-demographics* | | | | | | |
| Number of adults | -1.2991 (-1.726) | -0.5979 (-2.668) | | | -0.3694 (-1.903) | |
| No children | | -1.3784 (-5.063) | | | | |
| Presence of very young children | | | | | -0.6409 (-1.550) | |
| Presence of young children | | | | 1.0156 (4.070) | | |
| Presence of old children | | | | | 0.9832 (2.688) | |
| Female with very young children | -3.0788 (-3.291) | -1.2489 (-2.446) | -2.1106 (-2.592) | | -1.8622 (-2.700) | |
| More than one worker | | 0.601 (2.137) | 0.5729 (1.799) | | | |
| Number of drivers | 0.9598 (1.232) | | | | | |
| *Work-related characteristics* | | | | | | |
| Have more than one job | | -1.1408 (-2.142) | | -0.9847 (-2.523) | | |
| Part Time Job | 1.0869 (1.802) | | -1.7695 (-3.038) | | | |
| Distance to work < 2 miles | | | | | 0.7032 (2.170) | |
| *Mobility and situational characteristics* | | | | | | |
| Trip was made alone | | | | -0.4622 (-1.842) | -0.7167 (-1.612) | |
| Friday | | | | | -1.1507 (-1.878) | |
| *Household location variables* | | | | | | |
| Urban size < 1 million | | | | | 0.5003 (2.042) | |
| Urban size > 1 million with access to subway or rail | 0.7347 (1.390) | | | | | |
| *Baseline preference constants* | -10.6954 (-9.765) | -10.0333 (-18.504) | -12.5283 (-36.203) | -11.7083 (-46.243) | -10.7257 (-22.545) | |
| *Satiation Parameters ($\gamma$)* | 2.4941 (4.570) | 0.666 (2.914) | 2.7208 (7.426) | 0.9664 (5.345) | 3.2222 (8.009) | |

*2.3.3.1 Latent choice set generation component*

Estimation results for the latent choice set generation model component are presented in Table 2.3. In general, the model is found to offer plausible behavioral interpretations across a wide range of explanatory variables.

In the context of *individual characteristics*, it is found that older individuals are less likely to consider the before work period for undertaking non-work activities, perhaps a reflection of the greater household responsibilities that these individuals have, particularly in the early part of the day. Females, long known to shoulder a greater share of household maintenance responsibilities, are more likely to consider the work-to-home journey or the after work period for undertaking non-work activities. Cultural differences are observed with Asians less likely to consider work-based period for non-work activity engagement, while Hispanic men are disinclined to consider the home-to-work or work-to-home journey for undertaking non-work travel; these individuals possibly use alternative modes of transportation, which are less conducive to trip chaining non-work activities with the commute. More research on ethnicity based travel preferences might shed more light on this issue. Further, those without driver's license have an increased propensity to consider the non-auto alternative.

Among *household characteristics*, individuals in larger households are prone to considering the after-work period for undertaking non-work activities, a finding consistent with expectations. These households may have child care, spatial proximity, and vehicular availability constraints that motivate the scheduling of maintenance and recreational activities in the after-work period. Individuals in households with young children are less likely to use non-auto modes, presumably due to the need to transport small children easily. Individuals in households with school-age children, on the other hand, are more likely to consider the work-based period for non-work activities. This may be due to parents completing some errands while at work, due to the constraints associated with taking care of children after work. Greater vehicle availability is associated with consideration of the before work period, low income households tend to more inclined to include the non-auto alternative in their consideration choice set, and those in a medium income bracket tend to be less inclined to consider the before-work period for non-work activity engagement. Home owners are more likely to consider the work-to-home tour as an opportunity for undertaking non-work activities, presumably because such trip chaining brings about efficiencies in activity engagement.

Within *work-related characteristics*, individuals with flexible work schedules (individuals who reported that they can set their own start work time) appear to be more inclined to include the work-to-home journey in their consideration set. It is possible that individuals with flexible work hours work later schedules, thus leading to this result. People who have more than one job also appear to seek efficiency by considering trip chaining of non-work activities in the work-to-home tour, and avoiding consideration of the after-work period. Self-employed individuals appear to be more oriented towards seeking the pursuit of non-work activities in the early part of the day – before work, during the journey to work, or while at work. It is possible that these individuals have greater flexibility in the beginning part of the day and work a later schedule, thus eliminating consideration of later segments of the day for non-work activities. Individuals whose distance to work is less than two miles are more likely to consider the before work alternative compared to others. As these individuals live close to the work place, trip chaining is not likely to offer much efficiency gains. Moreover these individuals may bike or walk to work, thus necessitating the pursuit of non-work activities in the before work period.

Among the *mobility and situational characteristics*, variables indicating the number of bike and walk trips in the past week (a measure of the propensity to use non-motorized modes) are positively associated with consideration of the non-auto alternative. When the trip is undertaken alone, individuals are less likely to consider virtually all auto alternatives except for the work-to-home journey for undertaking non-work activities, presumably because solo errands are probably most efficiently accomplished on the way home from work. Mondays are usually the first day back at work after the weekend. It is possible that individuals are more tired on Mondays after work or have accomplished errands on the weekend days preceding the Monday. Individuals are more prone to consider the before-work period for non-work activities and less likely to consider the after-work period for a non-work activity. The positive coefficient associated with the before-work period may be a manifestation of the "Starbucks effect" (McGuckin *et al.*, 2005) being more pronounced on Mondays than other days of the week. On Fridays, individuals are less prone to consider the work-based period, perhaps because individuals can undertake non-work activities after the work period.

Only limited household *location variables* could be considered in this study due to the absence of detailed location information for households. Individuals that do not reside in an urban area are less likely to consider the after-work period possibly because their access to destinations is poorer in non-urban areas. Individuals residing in urban areas of size less than one million population are less likely to consider the work-based period for undertaking non-work activities. In addition, those who reside in larger areas with access to subway or rail, are less likely to consider the work-to-home tour for such activities, possibly because there is greater use of rail in these metro areas that deters trip chaining of non-work activities with the commute journey.

### *2.3.3.2 MDCEV model component*

Results of the MDCEV model component estimation are presented in Table 2.4. Younger individuals are found to be less inclined to pursue non-work activities while at work or in the after-work period. This finding is consistent with the notion that younger individuals may not have the household errands and child-related activities that would necessitate the pursuit of non-work activities during these periods. As the number of adults in a household increases, the likelihood of pursuing non-work activities in the before work period, during the home-to-work journey, or in the after-work period decreases, presumably due to greater household constraints. In addition, the ability to allocate tasks among multiple adult members reduces the need for each individual to pursue non-work activities. Chatman (2008) also found that increases in household size are associated with reductions in daily vehicle miles of travel.

The presence of children has important consequences for the participation in and mileage for non-work activities. Workers in households without children are less likely to participate in non-work activities during the home-to-work commute, presumably because they do not have to drop off children at school or day care on the way to work. Households with young children are less likely to pursue non-work activities in the after-work period, presumably because of child care constraints and the fact that very young children sleep early. Those with school age children, on the other hand are more likely to engage in non-work activities on the way home from work or in the after-work period, as they take care of household maintenance obligations and child-related activities. These findings are consistent with those reported in the literature by Rajagopalan *et al.* (2009), who found that the presence of young children induces more non-work stops on the journey home from work, and by Boarnet *et al.* (2004) who found that persons in households with more children accumulate more mileage for non-work activities. As expected, females with very young children are less likely to pursue non-work activities during all periods of the day, except on the journey home from work. This finding is consistent with the explanation that females with very young children are likely constrained by child-care responsibilities, and choose the journey home from work for accomplishing non-work errands for efficiency purposes.

Individuals in multiple worker households tend to accumulate more non-work activity engagement and mileage in the early part of the day, a finding consistent with that reported earlier by Strathman *et al.* (1994). It is possible that individuals in multiple worker households are more constrained in the after-work period and attempt to fulfill non-work activity needs in the early part of the day. Individuals in households with more drivers are more likely to accomplish non-work activities before work, possibly due to vehicle availability constraints.

Workers with more than one job are less inclined to pursue non-work activity engagement during the commute to and from work. This is consistent with expectations as these individuals are likely to be more constrained by multiple work schedules and cannot afford to undertake additional activities during the commute. Those with a part time job, on the other hand, are likely to have more time available in the before work period, thus motivating the pursuit of non-work activities in this period. On the other hand, they are less likely to pursue non-work activities in the work-based period, presumably because that period is shorter for them. When the distance to work is less than two miles, individuals are more likely to pursue non-work activities after work. Due to their proximity to the work place, these individuals might find it convenient to reach home and undertake an entirely separate tour for non-work activities. Also, these individuals may be using alternative modes of transportation that make trip chaining of non-work activities less convenient.

Explanations for the influence of mobility and situational characteristics are less intuitive. Workers traveling alone are less prone to undertake non-work activities during the journey from work to home or in the after-work period. The negative coefficient in the after-work period may be explained by arguing that non-work activities in this period tend to involve multiple household members. However, the negative coefficient associated with the work-to-home tour is more difficult to explain. It is possible that individuals tend to undertake solo non-work activities/errands during the earlier part of the day when other household constraints are not present. Another finding difficult to explain is the negative coefficient associated with Friday for the after-work period. One would expect this to have a positive coefficient as individuals are more inclined to undertake non-work activities on Friday after work. The variables related to the frequency of non-motorized travel in the past week were included to represent behavioral traits of inclination towards using non-motorized travel. The variable results in an expected effect on the choice process.

Workers living in urban areas with population less than one million are found to travel more miles for non-work activities during after-work time period, while those in larger urban areas with access to rail or subway travel more miles for non-work activities in the before work tour. The latter finding may be due to the higher prevalence of transit mode use for the journey to work, making it more challenging to couple non-work activities with the journey to and from work. All satiation parameters are statistically significant suggesting that there are substantial satiation effects in the pursuit of non-work activities and associated travel mileage for all periods of the work day.

## 2.4 Conclusions

The novel element added in this study is a first stage non-compensatory probabilistic choice set generation model that is capable of determining the consideration choice set for each individual as a function different attributes. In adding this component to the model system, it is explicitly recognized that not all the alternatives may be considered by the decision maker. The modeling approach recognizes that the choice set generation process is latent, or unobserved, to the analyst and therefore probabilistic in nature. While probabilistic choice set generation has been incorporated previously in the context of single discrete choice modeling situations, it has virtually never been accounted for in the context of a MDC modeling situation such as that considered in this study. The two-stage model system, including a probabilistic choice set generation component coupled with a MDCEV model component, is formulated and applied. This study offers a straightforward and practical approach for incorporating probabilistic latent choice set generation model components into activity-travel model systems.

The study of non-work activity-travel engagement has been a topic of much interest, both from the perspective of developing models to accurately forecast such travel and from the perspective of being able to implement transportation control measures that may help manage the demand for such travel. In this dissertation, non-work activity-travel engagement is modeled for workers considering various time-of-day blocks during which such activities can be undertaken. These time-of-day blocks constitute periods of the day defined around work schedules that invariably influence activity-travel patterns for employed individuals. The model system is applied to a travel survey sample belonging to the San Francisco Bay Area drawn from the 2009 NHTS of the United States. Both a two-stage model system including a latent choice set generation model component and a MDCEV model component, as well as a pure single-stage MDCEV model that assumes a constant (complete) choice set for all individuals, are estimated on the survey sample. A comparison of the measures of fit across the two model structures shows that the latent MDCEV specification offers vastly superior performance, thus pointing to the critical importance of considering latent choice set generation processes in the modeling of MDC choice decisions such as those considered here. The choice set generation model component clearly indicates that the consideration set of each individual is different, and highly dependent on a range of explanatory variables that describe the individual, household, and mobility and situational attributes.

The model results suggest that future travel survey collection efforts seriously consider eliciting choice set formation data, so that models may be based directly on the choice set formation information rather than using latent choice set generation approaches. In the absence of such data, our research results suggest that activity-based travel microsimulation model systems that purport to replicate individual and household activity-travel choices incorporate probabilistic latent choice set generation model components to fully capture the decision processes at play. Models estimated using simplistic and deterministic choice set generation rules, or assuming constant choice sets for all individuals, are inevitably going to provide inaccurate parameter estimates and consequently, poor forecasts of travel under a wide range of policy and socio-economic scenarios.

# CHAPTER 3: Introducing Multiple Constraints in the MDCEV Model

> The material in this chapter is draws substantially from the following published paper:
> Castro, M., C.R. Bhat, R.M. Pendyala and S.R. Jara-Diaz (2012) Accommodating multiple constraints in the multiple discrete-continuous extreme value (MDCEV) choice model. *Transportation Research Part B* 46(6), 729-743

In this chapter, the MDCEV model is extended to accommodate multiple resource constraints. In the next section the need to include multiple constraints is motivated, focusing on the consequences of ignoring such multiple constraints. In Section 3.2, the model is formulated using a flexible and general utility function form. The case with only inside goods is studied first, and then the case with inside and outside goods is presented. Section 3.3 presents the model application to time-use decisions, where individuals are assumed to maximize their utility from time-use in one or more activities subject to monetary and time availability constraints.

## 3.1 Multiple constraints in MDC choice models

An important assumption in the MDCEV model (as it stands currently) is that consumers maximize utility subject to a single linear binding constraint. The constraint is binding because the alternatives being considered are goods, and more of a good will always be preferred to less of a good; thus, consumers will consume at the point where the entire budget is exhausted. But in most choice situations, consumers usually face multiple resource constraints.[3] Some common examples of resource constraints relate to income

[3] The constraints included in our framework are structural constraints associated with limited resources. Psychological or personal barriers that limit consumption (such as personal tastes or beliefs) are included in the definition of the utility function, and are not modeled as constraints.

(or expenditure), time availability, and space availability, though other constraints such as rationing (for example, coupon rationing), energy constraints, technological constraints, and pollution concentration limits may also be active in other consumption choice situations. For instance, consumers' decisions regarding how they use their time in different activity purposes will naturally be dependent on both an income constraint (the expenditure incurred through participation in the different chosen activity purposes cannot exceed the money available for expenditure) and a time availability constraint (the time allocated to the various activities cannot exceed the available time). Another example relates to households' decisions regarding the quantity of purchase of grocery items. Here, in addition to the income constraint, there is likely to be a space constraint based on the household's refrigerating space or pantry storage space.

In multi-constraint situations, ignoring the constraints and considering only a single constraint can lead to utility preference estimations that are not representative of "true" consumer preferences. For example, consider the time-use of individuals with limited time and limited income. Also, assume that a water park in the area where the individuals live reduces service times (to get on water rides) as a promotion strategy to attract more patrons. This may relax the time constraints of the individuals as they make their participation choices. However, many of the individuals may still decide not go to the water park because of the income constraint they face. The net result would be that a model estimated only with a time constraint would not consider this income constraint effect and would underestimate the time-sensitivity of the individuals. Similarly, consider that the water park decides to reduce its admission fee. But individuals who are time constrained may still not be able to respond. In this case, the net result of ignoring the time constraint and using a single income constraint is an underestimation of the price sensitivity of the individuals. Further, the use of a single constraint in both these situations will likely lead to a poor data fit. The fundamental problem here is that there is a co-mingling of preference and constraint effects, leading to inconsistent preference estimation. Thus ignoring constraints will, in general, have serious negative repercussions for both model forecasting performance and policy evaluation.

To be sure, there has been earlier research in the literature considering multiple constraints (say *R* constraints), especially in the context of single discrete choice models. The basic approach of these studies, as proposed by Becker (1965) and sometimes referred to as a "full price" approach, essentially involves solving for (*R*-1) of the decision quantities (as a function of the remaining decision quantities) from (*R*-1) constraints, and substituting these expressions into the utility function and the one remaining constraint to reduce the utility maximization problem with multiple constraints to the case of utility maximization with a single constraint. Following Larson and Shaikh (2001), Hanemann (2006) provides a theoretical analysis of this problem for two and three constraints. Hanemman also defines an algorithm to represent the demand functions for multi-constraint problems, which requires an explicit form for the indirect utility function. However, in many cases the indirect utility function is unknown. In these cases, is required to solve a system of simultaneous equations, as would be the case with the direct utility function first-order conditions. Carpio *et al.* (2008) apply the "full price" in their model that includes the choice of an outside good and a single discrete choice from among all inside goods. Unfortunately, this single discrete choice-based approach is not easily extendable to the multiple discrete choice case because of the non-linearity of the utility expressions in the decision quantities. Even so, there is another problem with this approach. Specifically, there is an implicit assumption of the free exchangeability of constraints, which may not be valid because of the fundamentally different nature of the constraints. Thus, considering each constraint in its own right is a more direct and appealing way to proceed.

While there has been some research, even if limited, in the area of multiple constraints for single discrete choice models, the consideration of multiple constraints within the context of MDC econometric models has received scant attention (though there have been theoretical expositions of such a framework in the microeconomics and home production fields; see Jara-Díaz, 2007). The objective of this research is to contribute to this area by developing a multiple constraint extension of the MDCEV model. In doing so, a brief overview of two precursor studies of relevance is in order. The first study by Parizat and Shachar (2010) applied an MDC model with two constraints, based on a constant elasticity of substitution (CES) function with nonlinear pricing. Because Karush-Kuhn-Tucker (KKT) conditions are not sufficient for optimality with non-linear pricing, the estimation procedure is based on numerically locating the constrained optimal point, while taking all constraints into consideration. This is a substantial challenge, as acknowledged by Parizat and Shachar. They undertake the optimization using a simulated annealing algorithm after partitioning the solution space into regions. Of course, the approach obviates the need for a continuous, differentiable, and well-behaved utility function. But the approach loses the behavioral insights usually obtained from the KKT first-order conditions, and has to resort to a relatively "brute" force optimization approach rather than use analytic expressions during estimation. The second relevant study by Satomura *et al.* (2011) adopted a Bayesian approach to estimate an MDC model with multiple linear constraints. However, this effort:

1. Generalizes the restrictive linear expenditure system (LES) utility form used by Satomura *et al.*
2. Accommodates a random utility specification on all goods – inside and outside,
3. Is applicable to the case of inside goods only or inside and outside goods
4. Allows for the presence of any number of outside goods
5. Shows how the Jacobian structure has a closed-form structure for many MDC situations, which aids in estimation
6. Is applicable also to the case where each constraint has an outside good whose consumption contributes only to that constraint and not to other constraints.

## 3.2 Methodology

This section begins by considering two constraints – one being a monetary budget (or simply a "budget") constraint and the other being a time constraint. However, while the alternatives in the empirical analysis refer to activity purposes for participation over a fixed time period, for presentation ease, the alternatives in this section will be referred generally as goods. First, the case of inside goods with no outside goods is presented. Second, the case of outside goods and inside goods is introduced. Third, we formulate a related model in which each constraint has an outside good whose consumption contributes only to that constraint and not to others. Then, the model estimation is presented and identification considerations are discussed. Finally, in Section 3.2.5, we extend the analysis to include multiple (more than two) constraints.

3.2.1 Case of only inside goods

Consider Bhat's (2008) general and flexible functional form for the utility function that is maximized by a consumer subject to budget and time constraints:

$$\begin{aligned}
&\max U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\
&s.t. \quad \sum_{k=1}^{K} p_k x_k = E \\
&\qquad \sum_{k=1}^{K} g_k x_k = T \, ,
\end{aligned} \tag{3.1}$$

where the utility function $U(\boldsymbol{x})$ is the same defined in Equation (1.1). The first constraint in Equation (3.1) is the linear budget constraint, where $E$ is the total expenditure (or income) and $p_k > 0$ is the unit price of good $k$. The second constraint is the time constraint, where $T$ is the maximum time available and $g_k > 0$ is the unit time of good $k$. If modeling an incomplete demand system, the right hand variables in the constraints are not income and overall time, but realized total consumption money and time allocated to the current period. Then, $E$ and $T$ are a result of a predetermined allocation of resources to other goods.

In a time-use setting, $g_k$ =1 for all goods, since the decision variables $x_k$ themselves represent time investments, and one unit of time invested in an activity contributes exactly one unit toward $T$. However, in other choice contexts, there may be a unit-based contribution toward both constraints. For instance, when buying grocery items, each unit of a specific food item has a cost as well as may occupy a certain amount of space. Thus, purchases of food items will have to satisfy a budget constraint as well as a storage space constraint, based on unit prices as well as unit space needs (see Satomura *et al*., 2011). The formulation is derived including the notation $g_k$ to be general.

To find the optimal allocation of goods, the Lagrangian is constructed and the KKT conditions are derived. The Lagrangian function for the model of Equation (3.1) is:

$$L = U(\boldsymbol{x}) + \lambda\left(E - \sum_{k=1}^{K} p_k x_k\right) + \mu\left(T - \sum_{k=1}^{K} g_k x_k\right), \tag{3.2}$$

where $\lambda$ and $\mu$ are Lagrangian multipliers for the budget and time constraints, respectively. These values represent the marginal utility of expenditure and time. The KKT first order conditions for optimal consumption allocations ($x_k^*$) are:

$$\begin{aligned} &\frac{\psi_k}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k - \mu g_k = 0 \ \text{ if } x_k^* > 0,\ k = 1,2,...,K \\ &\frac{\psi_k}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k - \mu g_k < 0 \ \text{ if } x_k^* = 0,\ k = 1,2,...,K \end{aligned} \tag{3.3}$$

The optimal demand satisfies the conditions in Equation (3.1) and both constraints above. The budget and time constraints imply that only $K-2$ of the optimal consumptions $x_k^*$ need to be estimated because, given *E* and *T*, the quantity consumed of two goods is automatically determined from the quantity consumed for all other goods. Denote goods 1 and 2 as the goods to which the individual allocates non-zero consumption (the individual has to participate in at least 2 of the *K* purposes). The KKT conditions for these goods are:

$$\lambda + \mu h_1 = \frac{\psi_1}{p_1}\left(\frac{x_1^*}{\gamma_1}+1\right)^{\alpha_1-1}, \quad \lambda + \mu h_2 = \frac{\psi_2}{p_2}\left(\frac{x_2^*}{\gamma_2}+1\right)^{\alpha_2-1}, \tag{3.4}$$

where $h_k = g_k / p_k$, $p_k \neq 0$, $k = 1,2,...,K$. Solving the above equation system, the values of $\lambda$ and $\mu$ are given by:

$$\lambda = \frac{h_1\tilde{V}_2\psi_2 - h_2\tilde{V}_1\psi_1}{h_1 - h_2}, \quad \mu = \frac{\tilde{V}_1\psi_1 - \tilde{V}_2\psi_2}{h_1 - h_2}, \tag{3.5}$$

where $\tilde{V}_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k - 1}$ $(k=1,2,...,K)$. Substituting $\lambda$ and $\mu$ into Equation (3.4), the KKT conditions can be rewritten as:

$$\begin{aligned}
&\tilde{V}_k\psi_k = (1-\omega_k)\tilde{V}_1\psi_1 + \omega_k\tilde{V}_2\psi_2 \quad \text{if } x_k^* > 0,\ k=3,4,...,K \\
&\tilde{V}_k\psi_k < (1-\omega_k)\tilde{V}_1\psi_1 + \omega_k\tilde{V}_2\psi_2 \quad \text{if } x_k^* = 0,\ k=3,4,...,K,
\end{aligned} \tag{3.6}$$

where $\omega_k = \frac{h_1 - h_k}{h_1 - h_2}$.

The KKT conditions above have an intuitive interpretation. Note that, for any good $(k=1,2,...,K)$, $\tilde{V}_k\psi_k$ represents the marginal utility at the optimal consumption point $x_k^*$. The term $\omega_k$ ($k=3,4,...,K$) serves as a unique adjustment that applies to the marginal utilities of the chosen goods 1 and 2 in the $k^{th}$ good's KKT conditions. Specifically, $\omega_k$ takes account of the fact that it is not only the marginal utilities of goods (based on the preferences of the consumer) that play into the optimal consumptions, but also the unit prices $p_k$ and unit times $g_k$ of the goods. That is, $\omega_k$ serves the role of a price-time normalization involving the marginal utilities of the first two goods and good *k* ($k=3,4,...,K$). To illustrate, consider the case when $h_k = h_2$, which in the context of our time-use application corresponds to $p_k = p_2$ (since $g_k = 1\ \forall k$). Then, $\omega_k$ takes the value of one. The KKT conditions for this good *k* then state that good *k*'s optimal consumption will either be:

1. Positive such that the marginal utility at this optimal point is exactly equal to the marginal utility of good 2 at good 2's optimal consumption point, or
2. Zero if the marginal utility at zero consumption for good *k* is less than the marginal utility of good 2 at good 2's optimal consumption point.

On the other hand, when $h_k = h_1$ (or $p_k = p_1$), the KKT conditions for good *k* state that the optimal consumption for good *k* will either be

1. Positive such that the marginal utility at this optimal point is exactly equal to the marginal utility of good 1 at good 1's optimal consumption point, or

2. Zero if the marginal utility at zero consumption for good $k$ is less than the marginal utility of good 1 at good 1's optimal consumption point.

For other values of $h_k$ not equal to $h_1$ or $h_2$, $\omega_k$ serves to normalize the marginal utilities of goods 1, 2, and $k$ ($k = 3,4,...,K$) to enforce the general notion that, for consumed goods, the price-time normalized marginal utilities are the same at the optimal allocations, while, for the non-consumed goods, the price-time normalized marginal utilities at zero consumption are lower than the price-time normalized marginal utilities at the optimal consumptions of the consumed goods.

### 3.2.2 Case of outside and inside goods

In this section, the case when there are Hicksian composite outside goods and inside goods is considered. This is easily handled with minor revisions to the framework discussed in Section 3.2.1. For ease in exposition, assume that there are two outside goods, good 1 and good 2 (however, the method proposed can handle as many outside goods as there are in a choice situation). If both of these outside goods are non-essential, the formulation is identical to that in Section 3.2.1. If both of these are essential, the formulation needs modification and actually simplifies compared to that in Section 3.2.1. If one of these is non-essential, and the other is essential, the formulation entails a simple modification from the case when both are essential. In this section, we present the case when both the goods are essential. Modifications to the case of more than two outside goods and combinations of essential and non-essential outside goods are also discussed.

As discussed previously, at least two goods have to be chosen when individuals face two constraints. Assume also that there is a minimum consumption for outside good 1, given by $\gamma_1$ (the case of no minimum consumption becomes a special case with $\gamma_1 = 0$). Similarly, assume that there is a minimum consumption of good 2, given by $\gamma_2$. Following the notation used in Section 3.2.1, the utility maximization problem is:

$$\max U(\boldsymbol{x}) = \frac{\psi_1}{\alpha_1}\left[(x_1-\gamma_1)^{\alpha_1}-1\right]+\frac{\psi_2}{\alpha_2}\left[(x_2-\gamma_2)^{\alpha_2}-1\right]$$
$$+\sum_{k=3}^{K}\frac{\gamma_k}{\alpha_k}\psi_k\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k}-1\right)$$
$$s.t. \quad \sum_{k=1}^{K} p_k x_k = E \qquad (3.7)$$
$$\sum_{k=1}^{K} g_k x_k = T\ .$$

In the above model, $\gamma_k > 0$ for all $k$ is required. Also, $x_1-\gamma_1>0$ and $x_2-\gamma_2>0$ is needed. The result of the utility specification above is that an amount equal to $\gamma_1$ for the first good, and $\gamma_2$ for the second good, is first allocated to the two outside goods. Satiation effects for these first two goods start to "kick-in" only beyond these minimum consumption levels, at which point the usual satiation-based allocation mechanism sets in to determine consumption levels beyond the minimum quantities for the outside good, and the consumption levels of other inside goods. Since the $\gamma_k$ and $\alpha_k$ parameters serve very different roles for the outside goods, they are both theoretically estimable. However, because of the highly non-linear nature of the optimization problem, it is not uncommon to normalize some or all of these parameters to gain stability. A common normalization used in earlier MDC choice studies is to set $\alpha_k = 1$ (*i.e.*, $\alpha_k \to 1$) as well $\gamma_k = 0$ for the outside goods.

Using the above formulation, one can go through the same procedure as in the previous section. All expressions provided in the previous section remain valid, with the following substitutions: $\tilde{V}_1 = \frac{1}{p_1}\left(x_1^* - \gamma_1\right)^{\alpha_1-1}$, $\tilde{V}_2 = \frac{1}{p_2}\left(x_2^* - \gamma_2\right)^{\alpha_2-1}$ and $\tilde{V}_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1}$ $(k = 3,4,..., K)$. In the case of say three essential outside goods (say the first, second, and third goods), the expressions in the previous section again remain unchanged except that in addition to the substitutions for $\tilde{V}_1$ and $\tilde{V}_2$, we now also have $\tilde{V}_3 = \frac{1}{p_3}\left(x_3^* - \gamma_3\right)^{\alpha_3-1}$. In the case that the first outside good is an essential good, but not the second and third, the expressions in the previous section hold except that $\tilde{V}_1 = \frac{1}{p_1}\left(x_1^* - \gamma_1\right)^{\alpha_1 - 1}$ and $\tilde{V}_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k} + 1\right)^{\alpha_k - 1}$ $(k = 2,3,...,K)$. In this way, any number of outside goods (and any combination of essential and non-essential outside goods) can be accommodated.

3.2.3 Model with constraint-specific outside goods

In this section, we consider the case with two outside goods, denoted as the first and second goods. Let the first good be the numeraire good with respect to the budget constraint, so that $p_1 = 1$ and it does not appear in the time constraint ($g_1 = 0$). Let the consumption of the first good be denoted by $e_1$ in monetary units. Similarly, let the second good be the numeraire good with respect to the second constraint, so that $g_2 = 1$ and it does not appear in the budget constraint ($p_2 = 0$). Let the consumption of the second good be denoted by $t_2$ in time units. For instance, in the case of time-use, one may use savings as the first good (this has no time investment) and in-home leisure as the second good (this has no expenditure). Assume also that there is a minimum consumption for good 1, given by $\gamma_1$ (the case of no minimum consumption becomes a special case with $\gamma_1 = 0$). Similarly, assume that there is a minimum consumption of good 2, given by $\gamma_2$. Such a situation cannot immediately be handled by the framework in Section 3.2.2, because $h_2 = g_2 / p_2$ becomes undefined for the second alternative (and formulating the constraints in a form that uses the unit price in the numerator and the unit time in the denominator will not work either because the corresponding value is undefined for the first alternative).

Following the notation used in Section 3.2.2, the utility maximization problem is:

$$\max U(\boldsymbol{x}) = \frac{\psi_1}{\alpha_1}\left[(x_1-\gamma_1)^{\alpha_1}-1\right]+\frac{\psi_2}{\alpha_2}\left[(t_2-\gamma_2)^{\alpha_2}-1\right] + \sum_{k=3}^{K}\frac{\gamma_k}{\alpha_k}\psi_k\left(\left(\frac{e_k}{p_k\gamma_k}+1\right)^{\alpha_k}-1\right)$$
$$s.t. \quad \sum_{k=3}^{K} p_k x_k + e_1 = E \qquad \sum_{k=3}^{K} g_k x_k + t_2 = T\,. \tag{3.8}$$

The Lagrangian function for the model of Equation (3.8) is:

$$L = U(\boldsymbol{x}) + \lambda\left(E-\left(\sum_{k=3}^{K} p_k x_k + e_1\right)\right) + \mu\left(T-\left(\sum_{k=3}^{K} g_k x_k + t_2\right)\right). \tag{3.9}$$

Following the same procedure as for inside goods, the KKT conditions remain valid, with the following substitutions: $\tilde{V}_1 = (e_1^* - \gamma_1)^{\alpha_1 - 1}$, $\tilde{V}_2 = (t_2^* - \gamma_2)^{\alpha_2 - 1}$ and $\tilde{V}_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1}$ for $k = 3,4,...,K$.

<u>3.2.4 Model estimation</u>

Defining the baseline utility $\psi_k$ as in Equation (1.8), the KKT conditions of Equation (3.6), after some algebraic manipulations, are equivalent to:

$$\begin{aligned}
&\ln\tilde{V}_k + \boldsymbol{\beta}'\boldsymbol{z}_k + \varepsilon_k = \ln\left((1-\omega_k)\tilde{V}_1 \mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} + \omega_k\tilde{V}_2\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}\right) \text{ if } x_k^* > 0,\ k = 3,4,...,K \\
&\ln\tilde{V}_k + \boldsymbol{\beta}'\boldsymbol{z}_k + \varepsilon_k < \ln\left((1-\omega_k)\tilde{V}_1 \mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} + \omega_k\tilde{V}_2\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}\right) \text{ if } x_k^* = 0,\ k = 3,4,...,K\,.
\end{aligned} \tag{3.10}$$

Let $W_k \mid (\varepsilon_1, \varepsilon_2) = \ln\left((1-\omega_k)\tilde{V}_1\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} + \omega_k\tilde{V}_2\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}\right) - \ln\tilde{V}_k - \boldsymbol{\beta}'\boldsymbol{z}_k$, $k = 3,4,...,K$.

Under the assumptions that the unobserved terms $\varepsilon_k$ are independently distributed across all alternatives $(k = 1,2,...,K)$ and independent of $\boldsymbol{z}_k$, and follow a standard extreme value distribution with scale parameter $\sigma$, the probability that the individual chooses the first $M$ of the $K$ goods $(M \geq 3)$, given $\varepsilon_1$ and $\varepsilon_2$, is:

$$P\left(x_1^*,...,x_M^*,0,...,0 \,|(\varepsilon_1,\varepsilon_2)\right)=$$
$$\left\{\prod_{m=3}^{M}\frac{1}{\sigma}g\left(\frac{W_m\,|(\varepsilon_1,\varepsilon_2)}{\sigma}\right)det(J)\,|\,(\varepsilon_1,\varepsilon_2)\right\}\times\left\{\prod_{l=M+1}^{K}G\left(\frac{W_l\,|(\varepsilon_1,\varepsilon_2)}{\sigma}\right)\right\}, \qquad (3.11)$$

where $g$ is the standard extreme value density function, $G$ is the standard extreme value cumulative distribution function, and $det(J)\,|\,(\varepsilon_1,\varepsilon_2)$ is the determinant of the Jacobian $J$ with elements $J_{in}=\dfrac{\partial\varepsilon_{i+2}}{\partial x_{n+2}}$ $(i,n=1,2,...,M-2)$ conditional on the error terms of the first two alternatives. The first component on the right-hand side of Equation (3.11) involves the density of the $(M-2)$ chosen alternatives based on a change-of-variable calculus (the transformation from the random utility errors ($\varepsilon_m$, $m=3,4,...,M$) to the consumptions ($x_m$, $m=3,4,...,M$) generates the Jacobian $J$; the first and second alternatives do not appear in this term because they can be derived from the consumption of the other goods). The second component on the right-hand side of Equation (3.11) involves the probability of the goods that are not consumed ($M+1, M+2,...,K$). This is obtained by integrating $(\varepsilon^*_{M+1},\varepsilon^*_{M+2},...,\varepsilon^*_K)$ over the region consistent with no-consumption, based on the KKT inequalities in Equation (3.10). The determinant of the Jacobian takes different forms depending on the type of outside goods in the problem.

*3.2.4.1 Only inside goods case and the outside and inside goods case*

The determinant of the Jacobian conditional on $\varepsilon_1$ and $\varepsilon_2$ in the cases of (a) only inside goods and (b) outside and inside goods, has the following closed form (see Appendix B for the derivation):

$$det(J)\,\big|\,(\varepsilon_1,\varepsilon_2)=\left[\prod_{m=3}^{M}c_m\right]\left[1+\sum_{m=3}^{M}\frac{p_m b_m\big|(\varepsilon_1,\varepsilon_2)}{c_m}\right], \qquad (3.12)$$

where $c_m=\dfrac{1-\alpha_m}{x_m^*+\gamma_m}$ and $b_m\big|\,(\varepsilon_1,\varepsilon_2)=\dfrac{(1-\omega_m)\tilde{V}_1\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1}c_1+\omega_m\tilde{V}_2\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}c_2}{(1-\omega_m)\tilde{V}_1\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1}+\omega_m\tilde{V}_2\mathrm{e}^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}}$.

Integrating out the error terms $\varepsilon_1$ and $\varepsilon_2$ from Equation (3.11), the unconditional probability can be computed as:

$$P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = \int_{\varepsilon_1=-\infty}^{\infty} \int_{\varepsilon_2=-\infty}^{\infty} \left\{ \prod_{m=3}^{M} \frac{1}{\sigma} g\left( \frac{W_m \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) det(J) \,|\, (\varepsilon_1, \varepsilon_2) \right\}$$
$$\times \left\{ \prod_{l=M+1}^{K} G\left( \frac{W_l \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2, \tag{3.13}$$

where $f(\varepsilon_1)$ and $f(\varepsilon_2)$ refer to the extreme value density function with scale parameter $\sigma$. Finally, substituting the expression for the Jacobian from Equation (3.12) into the above equation, we obtain the expression below:

$$P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = \frac{1}{\sigma^{M-2}} \left[ \prod_{m=3}^{M} c_m \right] \int_{\varepsilon_1=-\infty}^{\infty} \int_{\varepsilon_2=-\infty}^{\infty} \left[ 1 + \sum_{m=3}^{M} \frac{p_m b_m \,|\, (\varepsilon_1, \varepsilon_2)}{c_m} \right]$$
$$\times \prod_{m=3}^{M} g\left( \frac{W_m \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) \times \left\{ \prod_{l=M+1}^{K} G\left( \frac{W_l \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2. \tag{3.14}$$

In the case when there is only one constraint (*i.e.*, $t_k = 0 \; \forall k$), the term $\omega_k$ is equal to zero for all goods. As a result, the KKT conditions from Equation (3.10) are equivalent to the traditional MDCEV's KKT conditions of Equation (1.6), and the term $b_m$ from the Jacobian is reduced to $c_1$. Then, the model collapses to the MDCEV with only one constraint. Thus, the multiple constraint MDCEV (MC-MDCEV) model in Equation (3.14) is the extension of the single constraint MDCEV model of Bhat (2008).

*3.2.4.2 Constraint-specific outside goods case*

Using the same assumptions on the error terms as earlier, the unconditional probability that the individual chooses the first $M$ of the $K$ goods ($M \geq 3$) is:

$$P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = \int_{\varepsilon_1=-\infty}^{\infty} \int_{\varepsilon_2=-\infty}^{\infty} \left\{ \prod_{m=3}^{M} \frac{1}{\sigma} g\left( \frac{W_m \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) det(J) \,|\, (\varepsilon_1, \varepsilon_2) \right\}$$
$$\times \left\{ \prod_{l=M+1}^{K} G\left( \frac{W_l \left|(\varepsilon_1, \varepsilon_2)\right.}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2, \tag{3.15}$$

The elements of the Jacobian are given by:

$$J_{in} \mid (\varepsilon_1, \varepsilon_2) = \frac{\partial \varepsilon_{i+2}}{\partial x_{n+2}} = p_{n+2}\left(a_{i+2} + b_{i+2}h_{n+2}\right) + \delta_{in}c_{i+2}, \quad i, n = 1,2,..., M-2 \tag{3.16}$$

where $a_{i+2} = \dfrac{\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} c_1}{\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} + \tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}}$, $\quad b_{i+2} = \dfrac{\tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2} c_2}{\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1+\varepsilon_1} + \tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}'\mathbf{z}_2+\varepsilon_2}}$, $\quad c_1 = \dfrac{1-\alpha_1}{x_1 - \gamma_1}$,

$c_2 = \dfrac{1-\alpha_2}{x_2 - \gamma_2}$, $c_k = \dfrac{1-\alpha_k}{x_k + \gamma_k}$ for $k = 3,4,..., K,$ and $\delta_{in} = 1$ if $i = n$ and $\delta_{in} = 0$ if $i \neq n$.

In this case, there is no closed-form structure for the determinant of the Jacobian, because of the presence of the $h_{n+2}$ term in the $i^{th}$ Jacobian element. But each element of the Jacobian may be constructed in a straightforward fashion based on the expressions above and then its determinant can be taken. If in the development above, $\alpha_k = 0$ for all $k$, $\gamma_1 = \gamma_2 = 0$, $\gamma_k = 1$ for $k = 3,4,..., K$, $\psi_1 = \psi_2 = 1$, and the error terms $\varepsilon_1$ and $\varepsilon_2$ (on the outside goods) are assumed not to exist (that is, their distributions collapse on zero), the result is Satomura *et al*.'s (2011) model.

3.2.5 Identification considerations

A couple of remarks about identification in the MC-MDCEV model are appropriate here. First, the scale parameter of the error terms $\sigma$ is always estimable in the case of the MC-MDCEV (at least from a theoretical standpoint), since $h_k$ cannot all be equal to 1 (if this were the case, the model would collapse to a single constraint MDCEV model). That is, when $h_k$ of at least two of the *K* goods are different, Equation (3.10) does not collapse in a way that can lead to non-identification of $\sigma$.

Second, as can be observed from the KKT conditions in Equation (3.10), it is not the case in the MC-MDCEV model that only differences in the $\boldsymbol{\beta}'\mathbf{z}_k$ terms matter. This is because the logarithm functional form operates on a function of the sum of quantities associated with the first two goods. However, note that the KKT conditions in Equation (3.10), as well as the probability expression in Equation (3.14), are essentially derived based on the consumption pattern of only $K-2$ goods, since the consumption of the first and second goods may be obtained by solving the two constraints once the consumption pattern of other goods is known. Thus, while the KKT conditions themselves (because of their functional form) do not impose any theoretical need for the normalization of constants and consumer-specific variables, it may be desirable to set the component of $\boldsymbol{\beta}'\boldsymbol{z}_k$ corresponding to these terms to zero for at least one of the first two goods.

### 3.2.4 Model with more than two constraints

Now consider the case with $R$ constraints and complete demand systems or the second stage of a two-stage incomplete demand systems. Each constraint is associated with a limited resource (money, time, space, *etc.*). To estimate the MDCEV model with $R$ constraints, individuals should consume at least $R$ goods from the choice set, and the maximization problem is given by Equation (3.17), where $a_k^r$ is the unitary contribution of good $k$ to constraint $r$ ($a_k^r \geq 0 \;\; \forall k = 1,2,...,K$, $\forall r = 1,2,...,R$) and $A^r$ is the total availability of resource $r$ ($\forall r = 1,2,...,R$).

$$
\begin{aligned}
&\max\, U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\
&s.t. \quad \sum_{k=1}^{K} a_k^1 x_k = A^1 \\
&\qquad\;\; \sum_{k=1}^{K} a_k^2 x_k = A^2 \\
&\qquad\qquad \vdots \\
&\qquad\;\; \sum_{k=1}^{K} a_k^R x_k = A^R .
\end{aligned}
\tag{3.17}
$$

This problem can be solved in the same way as for the case with two constraints, except that the probability expression for the consumption pattern will now involve $R$ integrals, one for each constraint. Modifications to cases Hicksian composite outside goods are similar to the two-constraint case.

### 3.3 Application to activity participation and time use modeling

3.3.1 Empirical Context

In the past decade and more, the activity-based approach to travel demand analysis has received much attention and seen considerable progress (see Pinjari and Bhat, 2010 and Ronald *et al.*, 2008 for recent reviews). A fundamental difference between the commonly used trip-based approach and the activity-based approach is the way time is conceptualized and represented in the two approaches. In the trip-based approach, time is reduced to being simply a "cost" of making a trip. The activity-based approach, on the other hand, treats time as an all-encompassing continuous entity within which individuals make activity/travel participation decisions. Thus, the central basis of the activity-based approach is that individuals' travel patterns are a result of their time-use decisions, which determine the generation and scheduling of trips. In this context, the empirical application in the current study contributes to the now growing number of utility-based micro-economic models of time-use (see Jiang and Morikawa, 2004, Bhat, 2005, Jara-Díaz, 2007, and Munizaga *et al.*, 2011).

3.3.2 Data description

The data source used for this analysis is obtained by combining two different disaggregate national survey data sets – the 2008 American Time Use Survey (ATUS) and the 2008 Consumer Expenditure Survey (CES).

The ATUS survey provides information on the amount of time individuals spend undertaking various in-home as well as out-of-home activities (such as work, study and recreational activities) on a pre-assigned day of the week. The data was collected through telephone interviews, and only individuals aged 15 years or older were eligible. The survey also obtained socioeconomic and demographic characteristics, the location of activities, and information on accompanying individual(s). For more details, the readers are referred to U.S. Bureau of Labor Statistics (2011a).

The CES survey provides data on the consuming and buying habits of households, both on a weekly basis (information is gathered based on two consecutive one-week survey periods) and over a longer period of time (information is gathered based on a quarterly period of expenditure). The survey includes information on small and frequent expenditures (such as grocery shopping, personal care, *etc*.) as well as larger and longer-term expenditures (household appliances, vehicles, *etc*.). Dollar amounts of the purchases (both goods and services) made during the survey period are recorded by the respondents irrespective of whether or not payment is made at the time of purchase. More detail can be found in U.S. Bureau of Labor Statistics (2011b).

For the current demonstration exercise to show the applicability of our proposed MC-MDCEV model, we used a combined and synthesized weekly time-use and expenditure data that Konduri *et al.* (2011) put together from the ATUS and the CES surveys. Since the ATUS collected time-use data at the individual level, while the CES survey obtained information at the household level, the analysis is confined to single individual households. A weekly analysis period is considered here because there is likely to be a weekly rhythm in time use and expenditure patterns (see Habib *et al.*, 2008). For full details of the synthesizing procedure and the scaling approach to a week's period from the ATUS daily time-use data and the CES weekly/quarterly data, the reader is referred to Konduri *et al.* (2011). Essentially, for the ATUS data, individuals who were surveyed on Sunday were chosen and time use patterns for Monday through Saturday were generated by appending records of individuals who reported time use patterns on other days of the week (based on matching on seven socioeconomic characteristics of interest – gender, age, employment status, race, college status, family income, and employment category). A weekly expenditure data set was constructed by applying a simple deflating factor approach on the CES quarterly data. The matching of the time-use and expenditure data was again undertaken based on a set of socioeconomic characteristics. The final sample used in the current empirical exercise includes the weekly time-use and expenditure patterns of 332 single individual households.

The decision variables used in this application are the weekly times allocated to different activities, measured in minutes. In the ATUS-CES sample developed by Konduri *et al.* (2011), 19 time use categories (by activity purpose) are defined, including work, study, personal business and care, shopping, social, entertainment and travel, separated by in-home and out-of-home activities. The weekly expenditures are categorized into 14 activity purposes, but they are not associated one-to-one with the time use categories. To apply our model, we need the time and expenditures for each alternative. Therefore, the time use activity purposes and the expenditure activity purposes are brought to a common classification taxonomy as follows:

1. Personal care (includes personal care, child care, healthcare, religious and spiritual activities and phone calls, considering both in and out home activities)
2. Eating out (includes all foods and drinks consumed out-of-home)
3. Leisure (in and out-of home social activities, recreation, sports, exercise and entertainment)
4. Shopping (both in and out home shopping activities)

The budget constraint represents limited purchasing power, and the unit price $p_k$ was computed for each alternative as the total expenditures (in U.S. dollars) divided by the total time allocated (in minutes) across all individuals. The time constraint represents time as a limited resource, bounded by the available time after performing mandatory activities, such as work and sleep. Since the decision variables themselves represent time investments, $g_k = 1 \, \forall k$.

Table 3.1 provides a summary of the final sample used in estimation. The time use by activity purpose shows that the first three alternatives are always chosen (the minimum time allocated is always greater than zero; that is, these three alternatives are "outside goods"). The final activity purpose, shopping, is selected by 97.3% of the individuals (that is, shopping is an "inside good"). The reason for these high levels of participation is the use of a weekly time frame. However, the presence of several outside goods does not pose problems because, as highlighted in Section 3.2.2, our proposed model can accommodate as many outside goods as there are in any choice context. The time use patterns in the different activity purposes in Table 3.1 indicate that individuals spend a substantial amount of time on leisure (about 29 hours per week, or 4 hours per day, on average), followed by personal care (about 8.5 hours per week, or 1.2 hours per day, on average). Shopping and eating out, on the other hand, are activity purposes in which individuals generally expend less time. These results are generally consistent with the associated unitary costs: leisure and personal care are the least expensive activities, while the most expensive ones are shopping and eating out. Even this preliminary data analysis suggests that individuals may not only be constrained by time, but also by income.

Table 3.1: Alternatives characteristics

| | | **Mean** | **Std. Dev.** | **Minimum** | **Maximum** |
|---|---|---|---|---|---|
| Personal care | Time Use [min] | 511.1 | 269.8 | 95 | 1931 |
| Eating Out | | 258.3 | 137.8 | 13 | 752 |
| Leisure | | 1750.0 | 488.3 | 561 | 3592 |
| Shopping | | 141.5 | 114.2 | 0 | 525 |
| Personal care | Unitary cost [US$/min] | 0.055 | 0.135 | 0.004 | 1.943 |
| Eating Out | | 1.340 | 3.093 | 0.012 | 43.393 |
| Leisure | | 0.117 | 0.203 | 0.001 | 2.331 |
| Shopping | | 1.922 | 5.398 | 0.008 | 64.172 |

Information on the independent variables is provided in Table 3.2. The sample has a slightly higher proportion of males relative to females, and the expected higher share of individuals of Caucasian origin (this includes individuals with a Hispanic background). Given that all individuals in the sample are employed, the percentage of students (both full and part time) is low. Following the definitions made by the U.S. Census Bureau (2010), information regarding the geographic area where the individuals live is also provided, including Midwest, South, and West and Northeast. The age range in the sample is between 20 and 64 years. The average number of hours worked per week is 41.8, which is a little higher that the standard five eight-hour days (almost 10% of the workers work more than 55 hours per week). Finally, the average weekly income is US$1,048, which roughly translates to an annual household income of about $54,500.

Individual socio-demographics and work-related characteristics were considered in the analysis. Socio-demographics capture the generic contextual and preference differences across individuals, while work-related characteristics capture the effects of more specific work schedules and time flexibility related attributes. In addition, we also considered interaction effects among the two sets of variables. The final variable specification was based on a systematic process of removing statistically insignificant variables and combining variables when their effects were not significantly different.

Table 3.2: Explanatory variables characteristics

| Discrete Variables | Sample Share [%] | | | |
|---|---|---|---|---|
| *Gender* | | | | |
| Male | 53.9 | | | |
| Female | 46.1 | | | |
| *Race* | | | | |
| Caucasian | 77.1 | | | |
| African American | 19.6 | | | |
| Other | 3.3 | | | |
| *Student status* | | | | |
| Student | 6.3 | | | |
| Not a student | 93.7 | | | |
| *Geographic region* | | | | |
| Midwest | 27.4 | | | |
| South | 34.6 | | | |
| West and Northeast | 38.0 | | | |
| **Continuous Variables** | **Mean** | **Std. Dev.** | **Minimum** | **Maximum** |
| Age [years] | 43.7 | 12.0 | 20.0 | 64.0 |
| Hours worked per week | 41.8 | 11.2 | 1.5 | 69.5 |
| Weekly income [US$] | 1,048.2 | 795.0 | 188.6 | 5,393.4 |

As discussed in Section 3.2.2, we set the baseline preferences for the first good to zero due to stability considerations. A number of alternative model forms were explored for the $\alpha_k$ and $\gamma_k$ parameters, which are summarized in Table 3.3. The estimation profiles include:

1. Setting $\alpha_k$ to zero for all goods, and estimating the $\gamma_k$ values (the $\gamma$ profile)
2. Setting $\alpha_k$ to zero for all goods, setting the $\gamma_k$ values to zero for the outside goods, and estimating the $\gamma_k$ values for the inside (shopping) good (the $\gamma^1$ profile)
3. Setting $\alpha_k$ to zero for all goods, setting the $\gamma_k$ values for the outside goods to the minimum consumptions as obtained from the descriptive statistics in Table 3.1 (the $\gamma^2$ profile), and estimate the $\gamma_k$ value for the inside good

4. Setting the $\gamma_k$ values to zero for the outside goods and one for the inside good (shopping), and estimating the $\alpha_k$ values (the $\alpha$ profile)
5. Setting the $\gamma_k$ values to the minimum consumptions, constraining $\gamma_k$ for shopping to 1, and estimating the $\alpha_k$ values (the $\alpha^1$ profile)
6. Setting the $\gamma_k$ values to zero for the outside goods, normalizing the $\alpha_k$ values for the inside goods to zero, and estimating the $\alpha_k$ values for the outside goods and the $\gamma_k$ value for the inside good (the $\alpha\gamma$ profile)
7. Setting the $\gamma_k$ values to the minimum consumptions for the outside goods, normalizing the $\alpha_k$ values for the inside goods to zero, and estimating the $\alpha_k$ values for the outside goods and the $\gamma_k$ value for the inside good (the $\alpha\gamma^1$ profile).

Table 3.3: Multiple constraint MDCEV estimation profiles

| Estimation profile | | $\alpha_k$ values | | $\gamma_k$ values | |
|---|---|---|---|---|---|
| | | **Outside goods** | **Inside goods** | **Outside goods** | **Inside goods** |
| 1 | $\gamma$ profile | 0 | 0 | √ | √ |
| 2 | $\gamma^1$ profile | 0 | 0 | 0 | √ |
| 3 | $\gamma^2$ profile | 0 | 0 | Min. consumption | √ |
| 4 | $\alpha$ profile | √ | √ | 0 | 1 |
| 5 | $\alpha^1$ profile | √ | √ | Min. consumption | 1 |
| 6 | $\alpha\gamma$ profile | √ | 0 | 0 | √ |
| 7 | $\alpha\gamma^1$ profile | √ | 0 | Min. consumption | √ |

*The symbol √ implies that the parameter is estimated*

While all of these profiles were estimable, for some of these profiles, we observed convergence and stability problems as manifested in large estimated standard errors. In any case, at the end, the $\gamma^1$ profile consistently emerged as the best among these alternative profiles as well as provided stable estimates, and is the one used in this study.

### 3.3.3 Model estimation results

In addition to the multiple constraint MDCEV (MC-MDCEV) model proposed in this research, we also estimated two single constraint MDCEV (SC-MDCEV) models in which only the time constraint is active or only the money constraint is active. The results of the time constrained MDCEV, money constrained MDCEV and MC-MDCEV models are presented in Table 3.4. The comparison of the results of the three models highlights two primary differences in variable effects. First, some variables are statistically insignificant in the SC-MDCEV models (gender in the money-constrained model and weekly income in both the singly constrained models), while they are statistically significant in the MC-MDCEV model. Second, the effects of variables on the choice process differ substantially across the two models, both in sign and magnitude.

Table 3.4: Multiple constraint MDCEV estimation results

| **Explanatory Variable** | **Time-Constrained MDCEV** | | **Money-Constrained MDCEV** | | **Multiple Constraints MC-MDCEV** | |
|---|---|---|---|---|---|---|
| | **Parameter** | **(t-stat)** | **Parameter** | **(t-stat)** | **Parameter** | **(t-stat)** |
| *Gender* | | | | | | |
| Males | | | | | | |
| Leisure | 0.4080 | (3.108) | 0.2008 | (0.902) | 0.5025 | (6.353) |
| *Geographic region* | | | | | | |
| West | | | | | | |
| Shopping | 0.3156 | (1.875) | 0.5561 | (1.893) | 0.2365 | (2.555) |
| *Hours worked per week* | | | | | | |
| Less than 35 hours | | | | | | |
| Shopping | 0.3676 | (2.319) | 0.7199 | (2.602) | 0.1951 | (2.812) |
| *Weekly income [US$]* | | | | | | |
| Less than 1,500 US$ | | | | | | |
| Eating Out | 0.0737 | (0.429) | 0.3260 | (1.149) | -0.9276 | (-124.134) |
| Baseline preference constants | | | | | | |
| Eating Out | -0.8245 | (-5.116) | -3.8588 | (-14.495) | -0.0440 | (-5.869) |
| Leisure | 1.0906 | (10.137) | -0.8021 | (-4.494) | 1.0228 | (17.080) |
| Shopping | -3.8492 | (-19.836) | -4.2361 | (-14.171) | -3.8724 | (-89.275) |
| Satiation Parameters ($\gamma$) | | | | | | |
| Shopping | 8.7343 | (4.942) | 4.2436 | (3.747) | 2.6158 | (32.664) |
| Scale Parameter ($\sigma$) | Not estimable | | 1.5722 | (39.313) | 0.4706 | (127.643) |
| Number of Parameters | 7 | | 8 | | 8 | |
| Log-likelihood at convergence | -6,639.1 | | -4,744.5 | | -3,018.7 | |
| Number of Observations | | | 332 | | | |

From a data fit standpoint, the log-likelihood measures for the SC-MDCEV models are -6,639.1 (time-constrained) and -4,744.5 (money-constrained), while the corresponding value for the MC-MDCEV model is -3,018.7. Although the improvement in log-likelihood measures of the MC-MDCEV model over the SC-MDCEV models is readily apparent, one can evaluate the models using the non-nested adjusted likelihood ratio test (see Equation (2.5)). For presentation ease, we focus on a comparison of the money-constrained SC-MDCEV (that provides a better fit than the time-constrained SC-MDCEV model) and the MC-MDCEV model proposed in this study. For this test, we use the base as the convergent log-likelihood value of the money-constrained MDCEV model with only the baseline constants and the shopping satiation parameter. This value, as shown in Table 3.4, is $L(c) = -4949.5$. Then, the adjusted likelihood ratio test is 0.0402 for the money-constrained MDCEV, while that for the MC-MDCEV model is 0.3889. The probability that the difference in the $\bar{\rho}_c^2$ values, which is 0.3889, could have occurred by chance is less than $\Phi\{-[-2 \times 0.3889 \times L(c)]^{0.5}\}$. This value is literally zero, indicating that the difference in adjusted rho-bar squared values between the two models is highly statistically significant and that the MC-MDCEV model is better from a data fit perspective.

In general, the MC-MDCEV offers plausible behavioral interpretations in the effects of exogenous variables. The gender effect indicates that men are more likely than women to participate in leisure activities. This result reinforce the stereotype of men being “glued to the tube”, a finding also observed in Habib *et al.* (2008) and Carrasco and Miller (2009). The influence of the geographic region of residence suggests that individuals living in the West region of the United States have a higher baseline preference for shopping. There is no obvious explanation for this finding, though the variable helps control for region-level differences in time-use patterns. Among the individual demographic variables, age, race and student status had no significant effects on time use.

The remaining two variables impacting the baseline preferences relate to work characteristics. Individuals who work less than 35 hours per week are more likely to shop than those who work more than or equal to 35 hours per week, possibly a reflection of time constraints that discourage participation in shopping, and a preference to participate in recreation and leisure activities after long workdays (see Goulias and Kim, 2001 for a similar result). Finally, low and middle income individuals (earning less than $1,500 per week) participate less in eat-out activities relative to their high income earning counterparts.

The baseline preference constants reflect the higher overall time investment in leisure and lower time investment in shopping and eat-out compared to personal care activities. The translation parameter for shopping allows corner solutions for that activity type.

Finally, because our model is based on constrained utility maximization, the Lagrangian multipliers may be gainfully employed to investigate the money value of time (*VT*). In particular, the multiplier $\lambda$ in Equation (3.2) is the marginal utility of income (it provides the increase in utility due to an increase in the expenditure constraint by one unit) and the multiplier $\mu$ is the marginal utility of time (it provides the increase in utility due to an increase in the available time by one unit). Thus, the implied *VT* is $\mu / \lambda$, which represents the willingness to pay to increase the available time *T* by one hour. This *VT* may be formulated as the ratio of the right-hand sides of the two expressions in Equation (3.5), and then estimated by integrating out the stochasticity embedded in the baseline utilities for the first two goods. The *VT* obtained is 62.18 US$/hour, a value that is similar to that obtained in Konduri *et al.* (2011). This *VT* value may be used for user benefits computations and social welfare analysis to evaluate the cost-benefits of investing in infrastructure improvements or in policies that have the effect of increasing participation in leisure and other non-work activities.

### 3.4 Conclusions

MDC choice models have gained attention in recent years to handle choice situations where consumers choose multiple alternatives simultaneously, along with a quantity dimension associated with the consumed alternatives. However, such models have been dominated by the assumption of a single linear resource constraint, which, when combined with consumer preferences, determines the optimal consumption point. In reality, consumers typically face multiple resource constraints such as those associated with time, money, and capacity. Ignoring such multiple constraints and instead using a single constraint can, and in general will, lead to poor data fit and inconsistent preference estimation, because there is a co-mingling of preference and constraint effects. In turn, this can have serious negative repercussions for both model forecasting performance and policy evaluation.

The purpose of this research is to extend the MDCEV model to accommodate multiple constraints. The formulation of the multiple constraints-MDCEV (MC-MDCEV) model uses a flexible and general utility function form, accommodates a random utility specification on all (inside and outside) goods, is applicable to the case of complete demand systems and incomplete demand systems (with outside goods that may be essential or non-essential), allows for the presence of any number of outside goods, shows how the Jacobian structure has a closed-form structure for many MDC situations, and is applicable also to the case where each constraint has an outside good whose consumption contributes only to that constraint and not to other constraints. Issues associated with identification are also discussed.

The proposed MC-MDCEV model is applied to time-use decisions, where individuals are assumed to derive their utility from participation in one or more activities within a fixed time interval and a monetary constraint. The sample for the empirical exercise was generated by combining time-use information from the 2008 ATUS with expenditure records from the 2008 U.S. CES. The estimation results show substantial differences across the MC-MDCEV and the MDCEV models in the estimated effects of variables. While it is difficult to definitively state that the parameter estimates from the MC-MDCEV model represent the "true" effects of variables, there is a clear suggestion that the MDCEV models are mis-estimated, given the vast improvement in data fit of the proposed MC-MDCEV model compared to the MDCEV models. Overall, the results strongly reinforce the notion that ignoring multiple constraints when present can have serious consequences for both forecasting purposes and welfare/policy analysis.

# CHAPTER 4: Allowing for Non-Additively Separable Utility Forms in the MDCEV Model

The purpose of this chapter is to generalize the MDCEV model to allow a non-additive utility structure. Section 4.1 motivates the need for a more flexible utility structure that accommodates rich substitution structures and complementarity effects, discusses the related work by Vásquez-Lavín and Hanemann (2008), and positions the current contribution. Section 4.2 clarifies the role of parameters in Vásquez-Lavín and Hanemann's non-additively separable utility function, identifies issues of theoretical consistency and restrictions that need to be maintained, and presents identification considerations. Section 4.3 provides the methodological framework to derive and estimate a new consistent MDCEV model that allows non-additively separable utility forms. Finally, Section 4.4 presents an empirical demonstration of the model proposed in this chapter.

## 4.1 Non-additively separable utility functions in MDC models

An additively separable (AS) utility function assumes that the marginal utility of one good is independent of the consumption of another good. This assumption has at least two important implications.

1. First, the marginal rate of substitution between any pair of goods is dependent only on the quantities of the two goods in the pair, and independent of the quantity of other goods. As indicated by Pollak and Wales (1992), this has consequences on the preferences directly. For example, let there be three food items: bread $(x_1)$, butter $(x_2)$, and peanut butter $(x_3)$. Consider an individual who tends to have bread and butter, or bread and peanut butter, but not bread alone. Such an individual may prefer the triplet [20,1,20] over [10,10,20], but may also prefer [10,10,1] over [20,1,1]. This violates additive utility, because, if the

individual prefers [20,1,20] over [10,10,20], s/he must prefer $[20,1,x_3]$ over $[10,10,x_3]$ according to additive utility. Consequently, the AS assumption substantially reduces the ability of the utility function to accommodate rich and flexible substitution patterns.

2. Second, the specification of a quasi-concave and increasing utility function with respect to the consumption of goods, along with additive utility across goods, immediately implies that goods cannot be inferior and cannot be complements (*i.e.*, they must be strict substitutes; see Deaton and Muellbauer, 1980, page 139). Besides, additive utility structure makes it difficult to recognize that consumers might have a preference for certain specific combinations of alternatives.

Overall, AS utility functions are substantially restricted in their ability to accommodate flexible dependencies (*e.g.*, complementarity and substitution) in the consumption of different goods).

To date, most MDC modeling frameworks, including the MDCEV model, have adopted an AS utility function. The exceptions are Song and Chintagunta (2007) and Mehta (2007), who accommodated complementarity and substitution effects in an MDC utility function to model purchase quantity decisions of house cleaning products. However, because of the model complexity, both studies use an indirect utility approach instead of a direct utility approach. As clearly articulated by Bunch (2009), the direct utility approach has the advantage of being closely tied to an underlying behavioral theory, so that interpretation of parameters in the context of consumer preferences is clear and straightforward. Further, the direct utility approach provides insights into identification issues. In this line of research, Lee and Allenby (2009) proposed a direct utility approach that incorporates a non-AS utility function. For this purpose, they grouped goods in categories assuming that goods in the same category are substitutes, while goods in different categories are complements. However, their modeling framework does not allow consumers to choose multiple goods within each category. Lee *et al.* (2010) proposed a direct utility model for measuring asymmetric complementarity.

Their model formulation accommodates both inside and outside goods, but it was only developed for the case of two goods.

Vásquez-Lavín and Hanemann (2008) extended Bhat's (2008) AS linear form and presented a quadratic version of it, allowing quadratic effects as well as allowing the marginal utility of each good to be dependent on the level of consumption of other goods. The quadratic form proposed by Vásquez-Lavín and Hanemann is a flexible functional form that has enough parameters to provide a second-order approximation to any true unknown direct twice-differentiable utility functional form. It also is a non-AS functional form. However, the flexibility is also a limitation, since the function can provide nonsensical results and be theoretically inconsistent for some combinations of the parameters and consumption bundles, an issue that has not received much attention in the literature (Sauer *et al.*, 2006). In fact, it is not possible to achieve global consistency (over all consumption bundles) in terms of the strictly increasing and quasi-concave nature of the utility function using the translog form. In the next section, we extend Vásquez-Lavín and Hanemann's discussion to clarify the role of parameters, identify issues of theoretical consistency and restrictions that need to be maintained, present identification considerations, and recommend a flexible form similar to the translog but that is easier to estimate and reduces global inconsistency problems associated with the translog.

## 4.2 Functional form in a non-AS utility function

4.2.1 The non-AS utility form

Vásquez-Lavín and Hanemann (2008) presented a quadratic version of the MDCEV utility form, as below:

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \left( \frac{\gamma_k}{\alpha_k} \left[ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] \left\{ \psi_k + \frac{1}{2} \sum_{m=1}^{K} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right] \right\} \right), \tag{4.1}$$

where $\psi_k > 0$, $\gamma_k > 0$, and $\alpha_k \leq 1$ for all *k*. The new interaction parameters $\theta_{km}$ allow quadratic effects (when $k = m$) as well as allow the marginal utility of good *k* to be dependent on the level of consumption of other goods. The model assumes symmetric interaction effects; that is, $\theta_{km} = \theta_{mk} \quad \forall\, k,m$. If $\theta_{km} = 0$ for all $k$ and $m$, the utility function collapses to MDCEV's linear form. If $\alpha_k \to 0 \quad \forall\, k$, the function collapses to the well-known direct basic translog utility function (see Christensen *et al*., 1975), and if $\alpha_k = 1 \quad \forall\, k$, we obtain the quadratic utility function used by Wales and Woodland (1983).

The marginal utility of consumption with respect to good $k$ is:

$$\frac{\partial U(\boldsymbol{x})}{\partial x_k} = \left(\frac{x_k}{\gamma_k} + 1\right)^{\alpha_k - 1} \left\{ \psi_k + \sum_{m=1}^{K} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right] \right\}. \tag{4.2}$$

The difference between the above expression and the MDCEV's utiliy is the presence of the second term in parenthesis, which includes the consumptions of other goods. Thus, the formulation is non-AS, but one in which the marginal utility of a good is dependent on the consumption amounts of other goods. The marginal utility at zero consumption of good $k$ collapses to:

$$\left.\frac{\partial U(\boldsymbol{x})}{\partial x_k}\right|_{x_k = 0} = \tilde{\pi}_k = \left\{ \psi_k + \sum_{m \neq k} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right] \right\}. \tag{4.3}$$

From above, it is clear that $\psi_k$ is no more the baseline (marginal) utility of good $k$ at the point of zero consumption of good $k$. Rather, it should be viewed as the baseline (marginal) utility of good $k$ at the point at which no good has yet been "consumed"; that is, when $x_m = 0 \quad \forall\, m$ (no consumption decision has yet been made). This also indicates that, if prices of all goods are the same, then the good with the highest value of $\psi_k$ will definitely see some positive consumption.

For the utility function to be strictly increasing, the condition $\tilde{\pi}_k > 0$ should be satisfied for all possible values of the consumption vector $\boldsymbol{x}$. This is in addition to the condition in the linear case where $\psi_k > 0 \quad \forall\, k$. The condition $\tilde{\pi}_k > 0$ is needed because we are considering the case of economic goods. In addition, a sufficiency condition for maintaining the decreasing marginal utility (or strict quasi-concavity) of the utility function is that the right-hand side of Equation (4.3) be a non-increasing function of $x_k$. The only way this condition will hold globally is if $\theta_{km} \geq 0$ for all *k* and *m*. The condition $\theta_{km} > 0$ implies that the goods *k* and *m* are complements (since the consumption of good *m* would increase the baseline marginal utility of good *k* and therefore consumption of good *k*). However, we would also like to allow rich substitution patterns in the utilities of goods by allowing $\theta_{km} < 0$ for some pairs of goods. As we discuss later, our methodology accommodates this, while also recognizing the constraint $\tilde{\pi}_k > 0$ ($k = 1,2,...,K$) during estimation and ensuring that it holds in the range of consumptions observed in the data.

#### *4.2.1.1 Parameter* $\gamma_k$

As in the linear case, the $\gamma_k$ parameter allows for corner solutions. In particular, the $\gamma_k$ terms shift the position of the point at which the indifference curves are asymptotic to the axes from $(0, 0, 0, ..., 0)$ to $(-\gamma_1, -\gamma_2, -\gamma_3, ..., -\gamma_K)$, so that the indifference curves strike the positive orthant with a finite slope. This, combined with the consumption point corresponding to the location where the budget line is tangential to the indifference curve, results in the possibility of zero consumption of good *k*.

In addition to allowing corner solutions, the $\gamma_k$ terms also serve as satiation parameters. However, unlike the linear case, $\gamma_k$ affects satiation for good *k* in two ways. The first effect is through the first linear term on the right-hand side of Equation (4.1), and the second is through the second term on the right-hand side of Equation (4.1) that generates quadratic effects. The overall effect depends on the sign and magnitude of the parameter $\theta_{kk}$ in the second term. If this term is negative, and particularly for high values of $\gamma_k$, we can get an inappropriate parabolic shape for the contribution of alternative *k* to overall utility within the range of $x_k$:

- On one hand, beyond a certain point of consumption of alternative *k*, there is negative marginal utility. This is because of the violation of the condition over $\tilde{\pi}_k$ in Equation (4.3). An illustration of the effect of $\gamma_k$ is provided in Figure 4.1, which plots the utility contribution of alternative *k* for different values of $\gamma_k$ ($\gamma_k$ = 1, 10, and 30), and $\psi_k = 1$, $\alpha_k \to 0$, $\theta_{kk} = -0.02$, and $\theta_{km} = 0 \; \forall \; m \neq k$. As can be observed, the utility curve for $\gamma_k = 30$ violates the requirement that the utility function be strictly increasing.

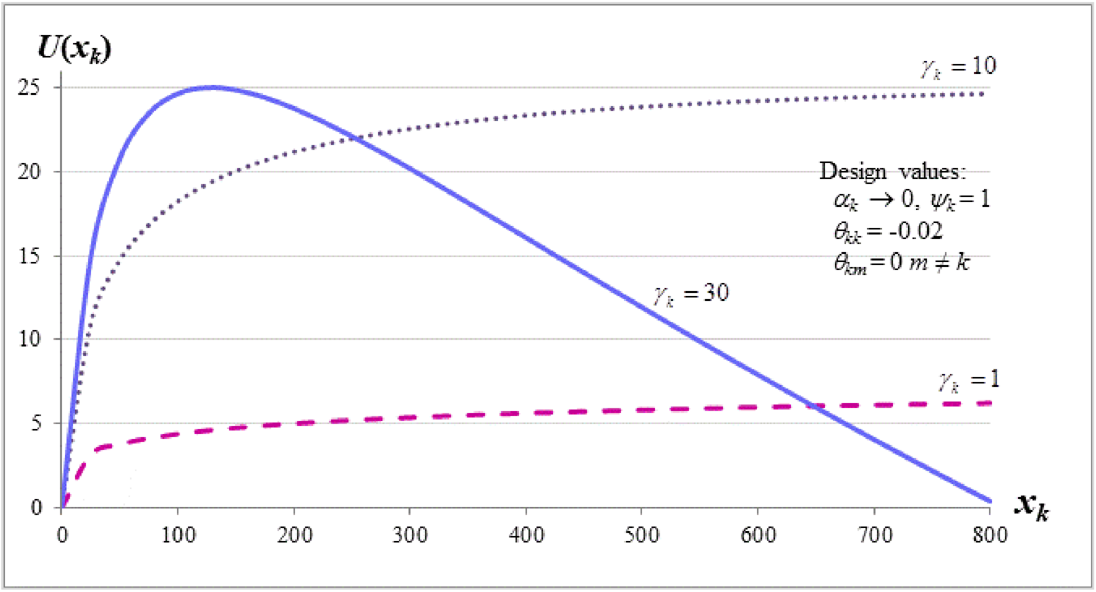


Figure 4.1: Effect of $\gamma_k$ on good *k*'s subutility function profile, negative $\theta_{kk}$

- On the other hand, if $\theta_{kk}$ is positive and quite high in magnitude, it is possible that, for high $\gamma_k$ values, there is in fact an increase in the marginal utility effect at low values of $x_k$ (essentially a violation of the strictly quasi-concave assumption of the utility function). This is because the right-hand side of Equation (4.1) becomes an increasing function of $x_k$ at low $x_k$ values. Figure 4.2 illustrates such a case for $\psi_k = 1$, $\alpha_k \to 0$, $\theta_{kk} = 0.2$, $\theta_{km} = 0 \; \forall \; m \neq k$, and different values of $\gamma_k$ ($\gamma_k$ = 1, 10, and 30). For $\gamma_k$ = 10, one can discern the increasing marginal utility until about 6.5 units after which the shape becomes one of decreasing marginal utility. The increasing marginal utility at low values is particularly pronounced for

$\gamma_k$ = 30, which continues until a value of 40 units before starting to decrease in marginal utility.

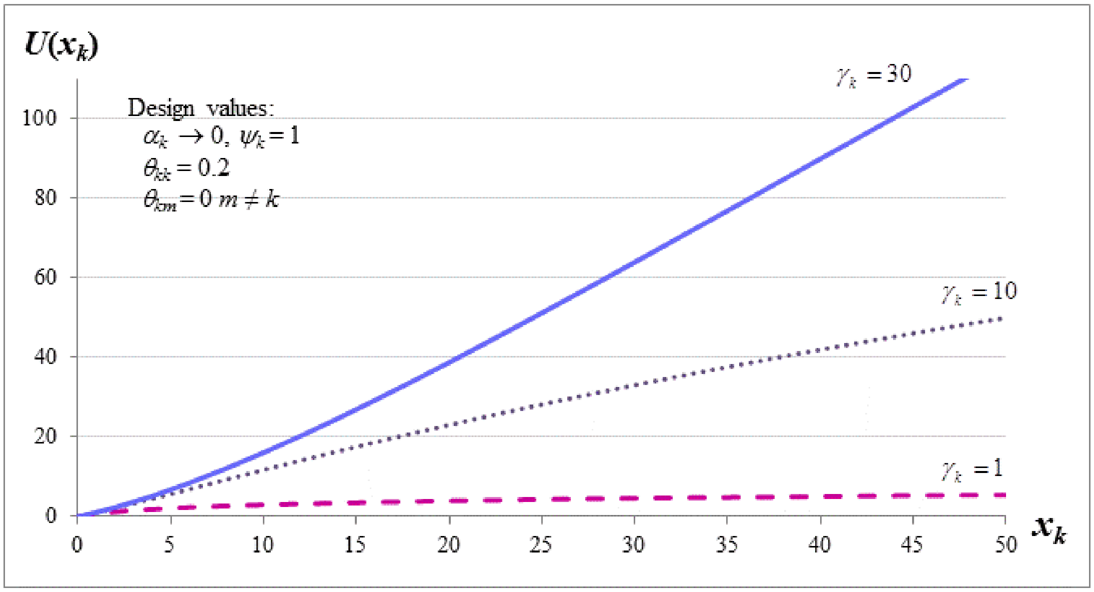


Figure 4.2: Effect of $\gamma_k$ on good *k*'s subutility function profile, positive $\theta_{kk}$

The translation parameters $\gamma_m$ of other goods also have an impact on the utility contribution of good *k*, through the influence on the baseline (marginal) utility of good *k* (see Equation (4.3)). Specifically, for a given value of $x_m$, the baseline (marginal) utility for good *k* increases as $\gamma_m$ increases for positive $\theta_{km}$ values and decreases as $\gamma_m$ increases for negative $\theta_{km}$ values.

### *4.2.1.2 Parameter $\alpha_k$*

The express role of $\alpha_k$ is to reduce the marginal utility with increasing consumption of good *k*; that is, it represents a satiation parameter. However, as in the case of the $\gamma_k$ effect on consumption of good *k*, there are two effects of the $\alpha_k$ parameter – one through the first linear term on the right-hand side of Equation (4.1), and the second through the quadratic effect caused by the combination of the first and second terms on the right-hand side of Equation (4.1).

The overall $\alpha_k$ effect depends on the sign and magnitude of the parameter $\theta_{kk}$ in the second term. If this term is negative, and particularly for values of $\alpha_k$ close to 1, we can get a "nonsensical" parabolic shape for the utility contribution of alternative $k$ within the usual possible range of $x_k$. An illustration is provided in Figure 4.3, which plots the utility contribution of alternative $k$ for $\gamma_k = 1$, $\psi_k = 1$, $\theta_{kk} = -0.03$, $\theta_{km} = 0 \; \forall \; m \neq k$, and different values of $\alpha_k$. As can be observed, for $\alpha_k = 0.06$ the utility profile violates the requirement that the utility function be strictly increasing. On the other hand, if $\theta_{kk}$ is positive and quite high in magnitude, it is possible that, for high $\alpha_k$ values, there is in fact an increase in the marginal utility effect at some low values of $x_k$. Figure 4.4 illustrates such a case for $\gamma_k = 1$, $\psi_k = 1$, $\theta_{kk} = 0.2$, $\theta_{km} = 0 \; \forall \; m \neq k$, and different values of $\alpha_k$. A non-conforming utility profile is obtained for $\alpha_k = 0.8$.

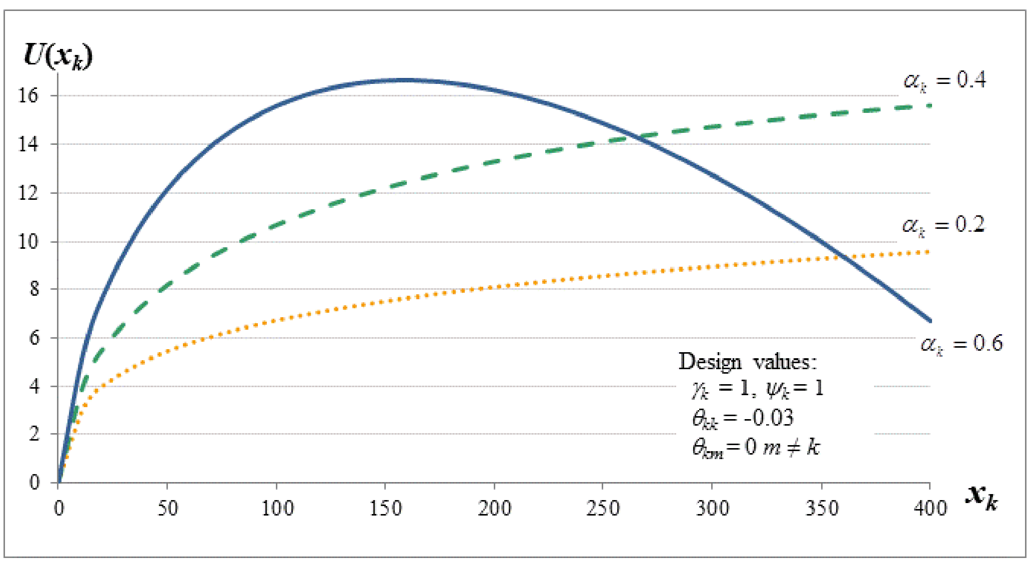


Figure 4.3: Effect of $\alpha_k$ on good $k$'s subutility function profile, negative $\theta_{kk}$

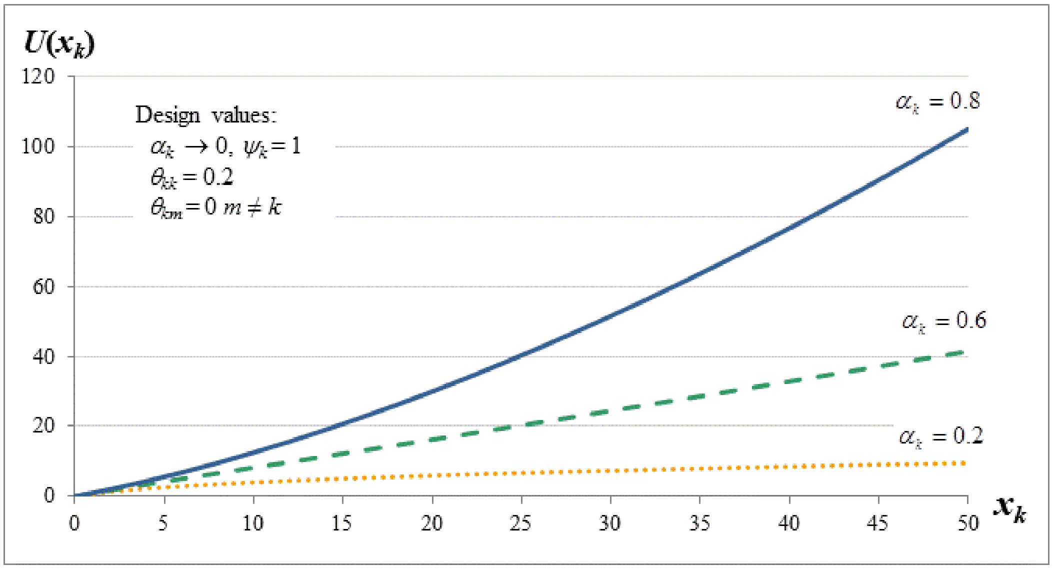


Figure 4.4: Effect of $\alpha_k$ on good *k*'s subutility function profile, positive $\theta_{kk}$

The $\alpha_m$ parameters for other goods also impact the baseline (marginal) utility of good *k* (see Equation (4.3)). For a given value of $x_m$, the baseline (marginal) utility for good *k* decreases as $\alpha_m$ falls down from 1 for positive $\theta_{km}$ values, and increases as $\alpha_m$ falls down from 1 for negative $\theta_{km}$ values.

4.2.2 Identification considerations

The total number of parameters in the flexible utility functional form of Equation (4.1) rises rapidly with the number of alternatives, especially in the $\theta_{km}$ terms ($k = 1,2,...,K$; $m = 1,2,...,K$). Assuming that $\theta_{km} = \theta_{mk}$ for all *k* (symmetric interaction effects), the number of parameters to estimate can be up to $K(K+1)/2$. There are also empirical identification issues that arise with the utility form. As in the linear case, empirically speaking, it is difficult to disentangle the effects of the $\gamma_k$ and $\alpha_k$ parameters for each good separately (see the discussion in Section 1.3.1).

In the case of the non-AS utility function, there is an additional empirical identification issue in both the $\gamma$ profile case and the $\alpha$ profile case. This is because the

$\theta_{kk}$ parameters in the quadratic utility functional form also serve as "satiation" parameters by providing appropriate curvature to the utility function. However, empirically speaking, it is difficult to disentangle the $\theta_{kk}$ effects from the $\gamma_k$ effects (for the $\gamma$ profile) and from the $\alpha_k$ effects (for the $\alpha$ profile) as long as the $\theta_{kk}$ effects do not become that negative as to bring on a parabolic shape at even low to moderate consumption levels (this latter case would anyway be inappropriate to represent the utility function):

- A utility profile based on a combination of $\theta_{kk}$ and $\gamma_k$ values for the $\gamma$ profile case can be closely approximated by a utility function based solely on $\gamma_k$ values with $\theta_{kk} = 0$. This is illustrated in Figures 4.5 for the $\gamma$ profile, with $\psi_k = 1$, $\alpha_k \to 0 \; \forall \, k$, and $\theta_{km} = 0 \; \forall \, m \neq k$. The figure shows that alternative $k$'s contribution to utility based on a certain combination of $\gamma_k$ and $\theta_{kk}$ values can be closely replicated by other combination values of $\gamma_k$ and $\theta_{kk}$. In particular, the utility profiles corresponding to combinations of $\gamma_k$ and $\theta_{kk}$ values can be replicated very closely by a profile that corresponds to $\gamma_k = 10$ and some specific $\theta_{kk}$ value, or by a profile that corresponds to $\theta_{kk} = 0$ and some specific $\gamma_k$ value.
- Similarly, a utility profile based on a combination of $\theta_{kk}$ and $\alpha_k$ values for the $\alpha$ profile case can be closely approximated by a utility function based solely on $\alpha_k$ values with $\theta_{kk} = 0$. This effect may be observed from Figure 4.6 for the $\alpha$ profile, where the utility profiles of different combinations of $\theta_{kk}$ and $\alpha_k$ values can be approximated closely by the profile corresponding to $(\alpha_k = 0.44, \theta_{kk} = 0)$ and $(\alpha_k = 0.5, \theta_{kk} = -0.01)$.

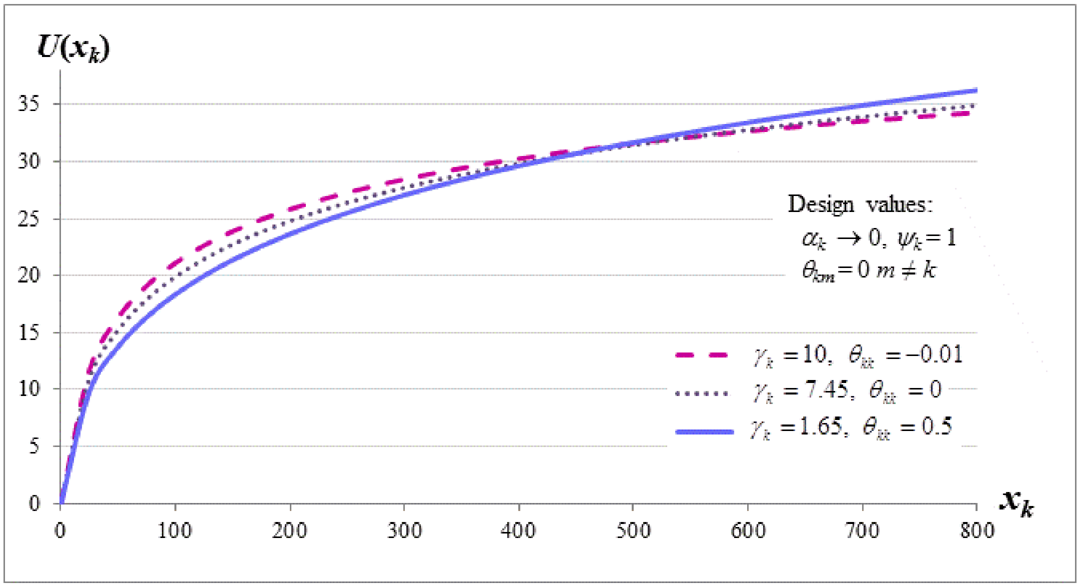


Figure 4.5: Alternative subutility profiles with different $\theta_{kk}$ and $\gamma_k$ values

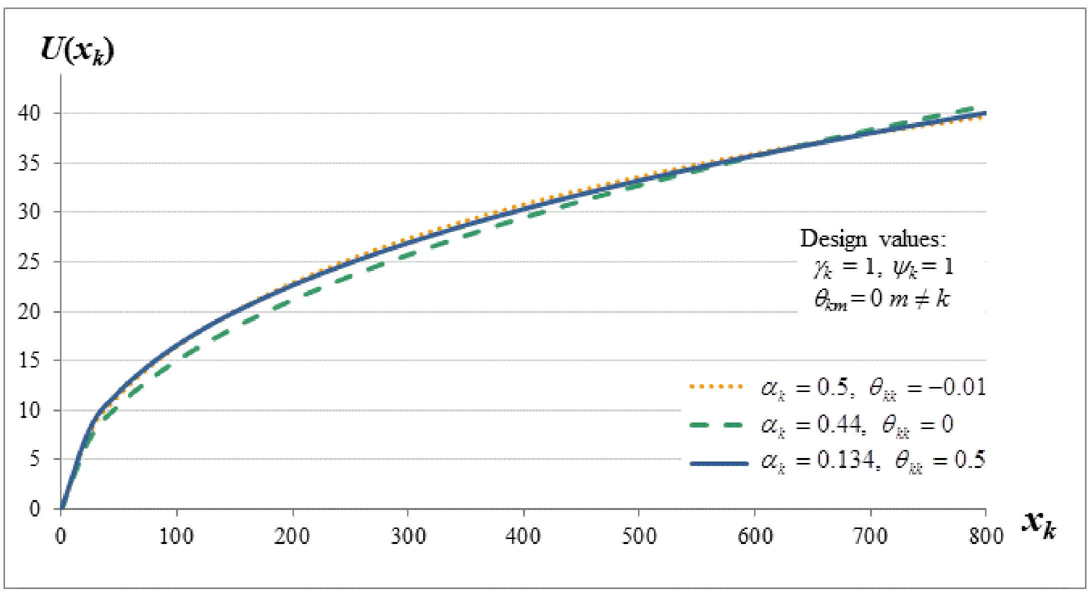


Figure 4.6: Alternative subutility profiles with different $\theta_{kk}$ and $\alpha_k$ values

The discussion above suggests that, without loss of empirical generality, one can normalize $\gamma_k = 1$ (and estimate $\theta_{kk}$) or set $\theta_{kk} = 0$ (and estimate $\gamma_k$) for each good *k* in the $\gamma$ profile case. In the $\alpha$ profile case, one can normalize $\alpha_k = 0$ (and estimate $\theta_{kk}$)

or set $\theta_{kk} = 0$ (and estimate $\alpha_k$) for each good *k* in the utility function. We propose to set $\theta_{kk} = 0$ for each good, since this immediately removes the possibility of a parabolic shape for the utility contribution of good *k*. At the same time, we immediately ensure that the marginal utility is strictly decreasing over the entire range of consumption values of the good *k*. In fact, the functional form proposed in this research remains within the class of flexible forms, while also retaining global theoretical consistency properties (unlike the translog and related flexible quadratic functional forms). The result is also clarity in the interpretation of the $\gamma_k$ and $\alpha_k$ parameters, which now have the same interpretation as satiation parameters corresponding to good *k* as in the linear utility function of the MDCEV model. Besides, the baseline marginal utility of good *k* now remains unchanged with the consumption of good *k*, which is intuitive.

### 4.3 Proposed non-AS utility function

Based on the discussion presented in Section 4.2, the following general formulation for the non-AS utility specification is proposed.

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \left( \frac{\gamma_k}{\alpha_k} \left[ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] \left\{ \psi_k + \frac{1}{2} \sum_{m \neq k} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right] \right\} \right) \tag{4.4}$$

In the above function, the analyst will need to set $\alpha_k \to 0 \, \forall \, k$ and estimate the $\gamma$ profile, or set $\gamma_k = 1$ and estimate the $\alpha$ profile as follows.

$$\begin{aligned} &\gamma \text{ profile} \qquad U(\boldsymbol{x}) = \sum_{k=1}^{K} \left( \gamma_k \ln\left( \frac{x_k}{\gamma_k} + 1 \right) \left\{ \psi_k + \frac{1}{2} \sum_{m \neq k} \theta_{km} \, \gamma_m \ln\left( \frac{x_m}{\gamma_m} + 1 \right) \right\} \right) \\ &\alpha \text{ profile} \qquad U(\boldsymbol{x}) = \sum_{k=1}^{K} \left( \frac{1}{\alpha_k} \left( (x_k + 1)^{\alpha_k} - 1 \right) \left\{ \psi_k + \frac{1}{2} \sum_{m \neq k} \frac{\theta_{km}}{\alpha_m} \left( (x_m + 1)^{\alpha_m} - 1 \right) \right\} \right). \end{aligned} \tag{4.5}$$

If an outside good is present, label the outside good as the first good which now has a unit price of one (*i.e.*, $p_1 = 1$). This good, being an outside good, has no interaction term effects with the inside goods; *i.e.*, $\theta_{1m} = 0 \; \forall m \, (k \neq 1)$. The utility functional form of Equation (4.4) now needs to be modified as follows:

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1}(x_1 + \gamma_1)^{\alpha_1}\psi_1 + \sum_{k=2}^{K}\left(\frac{\gamma_k}{\alpha_k}\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k}-1\right]\left\{\psi_k + \frac{1}{2}\sum_{\substack{m\neq k\\ m\neq 1}}\theta_{km}\frac{\gamma_m}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right]\right\}\right) \tag{4.6}$$

The consumer maximizes utility the as provided by Equation (4.4) or (4.6) subject to the budget constraint that $\sum_{k=1}^{K} p_k x_k = E$, where $p_k$ is the unit price of good *k* and *E* is total expenditure across all goods. The analyst can solve for the optimal consumption allocations $x_k^*$ by forming the Lagrangian and applying the Karush-Kuhn-Tucker (KKT) conditions.

The Lagrangian function for the problem is:

$$L = U(\boldsymbol{x}) - \lambda\left[\sum_{k=1}^{K} x_k p_k - E\right]. \tag{4.7}$$

where $\lambda$ is the Lagrangian multiplier associated with the budget constraint (that is, it can be viewed as the marginal utility of total expenditure or income). The KKT first-order conditions for the optimal consumption allocations (the $x_k^*$ values) are given by:

$$\begin{aligned}&\frac{\partial U(\boldsymbol{x})}{\partial x_k^*} - \lambda = 0, \text{ if } x_k^* > 0,\ k = 1,2,\ldots,K\\ &\frac{\partial U(\boldsymbol{x})}{\partial x_k^*} - \lambda < 0, \text{ if } x_k^* = 0,\ k = 1,2,\ldots,K\end{aligned} \tag{4.8}$$

To complete the econometric model, the analyst needs to introduce stochasticity. As in the MDCEV model, we maintain that a stochastic component must be included in the context of each alternative *k*, rather than ignoring the stochastic component for one of the alternatives. However, in the non-AS utility of Equation (4.4) two different formulations can be proposed, depending on the assumptions made over the source of the error term:

1. **Random utility-deterministic maximization (RU-DM) decision postulate**: in the first formulation, any stochasticity in the KKT conditions originates from the

analyst's inability to observe all factors relevant to the consumer's utility formation. Individuals are assumed to know all relevant factors impacting choice, and make an error-free maximization of overall utility (subject to the expenditure constraint) to determine their consumption patterns.

2. **Deterministic utility-random maximization or (DU-RM) decision postulate**: in the second formulation, not only is the consumer aware of all factors relevant to utility formation, but the analyst observes all of these factors too. However, consumers are assumed to make random mistakes (*errors*) in maximizing utility (subject to the expenditure constraint), which gets manifested in the form of stochasticity in the KKT conditions. This DU-RM decision postulate was explicitly identified by Wales and Woodland (1983) in their MDC formulation.

While the DU-RM postulate is seldom used for KKT models in the econometric literature, it certainly is a plausible one that should not be summarily dismissed. It also allows the usual computations of compensating variation for welfare analysis (a common reason for modeling consumer preferences) as does the RU-DM postulate.

In the MDCEV model (AS case), both the DU-RM and RU-DM decision postulates lead to exactly the same model. Since the two postulates are empirically indistinguishable, one can use either postulate to motivate the model. However, this ceases to be the case when moving from the AS utility form to the non-AS utility functional form of Equation (4.4) when random utility is specified through a multiplicative exponential error term on the $\psi_k$ term.

Additionally, we propose a formulation that combines these two formulations (RU-DM and DU-RM) in a **random utility-random maximization (RU-RM) decision postulate**, which is particularly convenient and general for the non-AS case. Intuitively, we are able to distinguish between random preferences and random maximization errors in the non-AS case because the former is associated with the "no-consumption" baseline (marginal) utilities that then remains fixed during the consumer's navigation through the optimization process, while the latter is essentially associated with overall mistakes represented by random errors in the baseline (marginal) utilities after including substitution/complementarity effects.

In the next three sections, we discuss the RU-DM, DU-RM and RU-RM formulations.

### 4.3.1 The RU-DM non-AS utility formulation and model

#### *4.3.1.1 Case of only inside goods*

Consider the following random utility form originating from the non-AS utility function form of Equation (4.4) for the no-outside good case:

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \left( \frac{\gamma_k}{\alpha_k} \left[ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] \left\{ \psi_k \exp(\xi_k) + \frac{1}{2} \sum_{m \neq k} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right] \right\} \right). \tag{4.9}$$

where $\xi_k$ is an independently and identically distributed (IID, across alternatives) random error term with a scale parameter of $\sigma$ ($\sigma$ can be normalized to one if there is no variation in the unit prices across alternatives). $\xi_k$ captures idiosyncratic (unobserved) characteristics that impact the baseline (marginal) utility of good $k$ at the point at which no expenditure outlays have yet been made on any alternative. This stochastic specification is quite intuitive, since it indicates an intrinsic (unobserved) individual preference for each alternative whose magnitude remains stable as the consumer navigates to reach her/his optimal expenditure point.

The KKT conditions then are (see Equation (4.8)):

$$\begin{aligned} &\eta_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda = 0, \text{ if } x_k^* > 0, \; k = 1,2,...,K \\ &\eta_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda < 0, \text{ if } x_k^* = 0, \; k = 1,2,...,K \end{aligned} \tag{4.10}$$

where $\eta_k = \psi_k \exp(\xi_k) + W_k$ and $W_k = \sum_{m \neq k} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[ \left( \frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right]$.

Now define $\omega_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1}$ and $L_k = \frac{1-\alpha_k}{p_k(x_k^*+\gamma_k)}$ for $k=1,2,...,K$, and let $R_k = \eta_1 \cdot \frac{\omega_1}{\omega_k} - W_k$. Let the first alternative be the one to which the consumer allocates some non-zero budget amount ($x_1^* > 0$). Then, the KKT conditions may be simplified as follows (note that $\psi_k = \exp(\boldsymbol{\beta}'\boldsymbol{z}_k)$):

$$\begin{aligned} \xi_k &= \ln(R_k \mid \xi_1) - \boldsymbol{\beta}'\boldsymbol{z}_k, \text{ if } x_k^* > 0,\ k=2,...,K \\ \xi_k &< \ln(R_k \mid \xi_1) - \boldsymbol{\beta}'\boldsymbol{z}_k, \text{ if } x_k^* = 0,\ k=2,...,K \end{aligned} \tag{4.11}$$

Next, assume that $g(.)$ and $G(.)$ are the standardized versions of the probability density function and standard cumulative distribution function characterizing $\xi_k$, repectively. Then, the probability that the individual allocates expenditure to the first $M$ of the $K$ goods may be derived to be:

$$\begin{aligned} P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = \int_{\xi_1=-\infty}^{\xi_1=+\infty} & abs\,(det(J \mid \xi_1))\left\{\left(\prod_{i=2}^{M}\frac{1}{\sigma}g\left[\frac{\ln(R_i \mid \xi_1) - \boldsymbol{\beta}'\boldsymbol{z}_i}{\sigma}\right]\right)\right\} \\ & \times\left\{\prod_{s=M+1}^{K} G\left[\frac{\ln(R_s \mid \xi_1) - \boldsymbol{\beta}'\boldsymbol{z}_s}{\sigma}\right]\right\}\frac{1}{\sigma}g\left(\frac{\xi_1}{\sigma}\right)d\xi_1, \end{aligned} \tag{4.12}$$

where $J \mid \xi_1$ is the Jacobian conditional on $\xi_1$, whose elements for goods $i,n = 1,2,...,M-1$ are given by: (see Appendix C for derivation):

$$\begin{aligned} J_{in} \mid \xi_1 = \frac{1}{R_{i+1} \mid \xi_1}\Bigg\{ & \frac{\omega_1}{\omega_{i+1}}\left[(\eta_1 \mid \xi_1)(p_1^2 L_1 + p_{i+1}L_{i+1}z_{in}) + \theta_{1,n+1}p_{n+1}\omega_{n+1}\right] \\ & + p_{n+1}\left[\theta_{1,n+1}p_1\omega_1 - \theta_{i+1,n+1}\omega_{n+1}(1-z_{in})\right] \Bigg\} \end{aligned} \tag{4.13}$$

In the above expression, $z_{in} = 1$ if $i = n$, and $z_{in} = 0$ if $i \neq n$.

The probability expression in Equation (4.12) is a simple one-dimensional integral, which can be computed using quadrature techniques. Note that the distribution for $\xi_k$ can be any univariate distribution, though the normal distribution may be convenient if there are also random normal coefficients in the $\boldsymbol{\beta}$ vector to capture unobserved individual heterogeneity (then the one-dimensional normal integral becomes simply a part of a multi-dimensional normal integration that can be evaluated using familiar simulation techniques). Such a random-coefficients specification allows a flexible covariance structure between the elements of the $\boldsymbol{\beta}$ vector, and can also include covariances among the baseline utilities of alternatives (as in a mixed multinomial logit structure). The model may be estimated using traditional maximum likelihood techniques.

Note, however, that two sets of conditions need to be considered. The first condition is that the marginal utility of any good at any point of consumption should be positive (for strictly increasing utility functions). This condition is met by setting $\tilde{\pi}_k > 0$ (see Equation (4.3)). The second set of conditions is that the term $R_k \mid \xi_1$ should always be positive (for each alternative *k*) as it inside the logarithmic function in Equation (4.11). While the first set of conditions need not be imposed explicitly (since the consumption point at which the marginal utility of a good becomes negative cannot be an optimal consumption point), it is important to ensure the second set of conditions to avoid estimation failures.

*4.3.1.2 Case of outside and inside goods*

When an outside good is present, the econometrics again simplify considerably. For the outside good (say, the first alternative), we have the following: $W_1 = 0$, $\boldsymbol{\beta}'\boldsymbol{z}_1 = 0$, $\psi_1 = 1$, $p_1 = 1$ and $\eta_1 = \exp(\xi_1)$. The random utility function originates from Equation (4.9) and takes the following form:

$$U(\boldsymbol{x}) = \frac{1}{\alpha_1}(x_1 + \gamma_1)^{\alpha_1}\exp(\xi_1)$$
$$+ \sum_{k=2}^{K}\left(\frac{\gamma_k}{\alpha_k}\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right]\left\{\psi_k \exp(\xi_k) + \frac{1}{2}\sum_{\substack{m \neq k \\ m \neq 1}}\theta_{km}\,\frac{\gamma_m}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m} - 1\right]\right\}\right). \tag{4.14}$$

The probability expression takes the same form as in Equation (4.12) with the following modifications to the $\omega_k$ terms: $\omega_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1}$ for $k = 2,...,K$ and $\omega_1 = (x_1^* + \gamma_1)^{\alpha_1-1}$. The Jacobian elements conditional on the error term $J_{in} \mid \xi_1$ ($i,n = 1,2,...,M-1$) are as follows:

$$J_{in} \mid \xi_1 = \frac{1}{R_{i+1} \mid \xi_1}\left\{\frac{\omega_1}{\omega_{i+1}}\left[(\eta_1 \mid \xi_1)(L_1 + p_{i+1}L_{i+1}z_{in})\right] - p_{n+1}\theta_{i+1,n+1}\omega_{n+1}(1-z_{in})\right\}. \quad (4.15)$$

<u>4.3.2 The DU-RM non-AS utility formulation and model</u>

*4.3.2.1 Case of only inside goods*

Following the notation of the previous chapters, the KKT conditions of Equation (4.8) take the form presented in Equation (4.16), where $\tilde{\pi}_k$ is the baseline marginal utility as provided in Equation (4.3).

$$\begin{aligned}
&\tilde{\pi}_k\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda = 0, \text{ if } x_k^* > 0,\ k = 1,2,...,K \\
&\tilde{\pi}_k\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda < 0, \text{ if } x_k^* > 0,\ k = 1,2,...,K.
\end{aligned} \quad (4.16)$$

Stochasticity may be introduced explicitly in the KKT conditions in the usual multiplicative exponential form as follows:

$$\begin{aligned}
&\tilde{\pi}_k \exp(\varepsilon_k)\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda = 0, \text{ if } x_k^* > 0,\ k = 1,2,...,K \\
&\tilde{\pi}_k \exp(\varepsilon_k)\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda < 0, \text{ if } x_k^* > 0,\ k = 1,2,...,K.
\end{aligned} \quad (4.17)$$

The optimal demand satisfies the conditions in Equation (4.17) and the budget constraint. The structure is now exactly the same as the MDCEV model. Specifically, consider an extreme value distribution for $\varepsilon_k$ and assume that $\varepsilon_k$ is independent of $\psi_k$, $\gamma_k$, and $\alpha_k$ ($k = 1,2,...,K$). The $\varepsilon_k$ terms are also assumed to be independently distributed across alternatives with a scale parameter of $\sigma$ ($\sigma$ can be normalized to one if there is no variation in unit prices across goods). In this case, the probability expression collapses to the following MDCEV closed-form:

$$P\left(x_1^*, x_2^*, ..., x_M^*, 0, ..., 0\right) = abs(det(J))\frac{1}{\sigma^{M-1}}\left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right](M-1)!, \qquad (4.18)$$

where $V_k = \ln(\tilde{\pi}_k) + (\alpha_k - 1)\ln\left(\frac{x_k^*}{p_k} + 1\right) - \ln p_k \quad (k = 1,2,...,K)$, and the elements of the Jacobian $J$ are given by: (see Appendix D for the derivation)

$$J_{in} = p_{n+1}\omega_{n+1}\left[\frac{\theta_{1,n+1}}{\tilde{\pi}_1} - \frac{\theta_{i+1,n+1}}{\tilde{\pi}_{i+1}}(1 - z_{in})\right] + p_1^2\omega_1\frac{\theta_{1,i+1}}{\tilde{\pi}_{i+1}} + p_{n+1}\left[p_1 L_1 + L_{n+1} z_{in}\right], \qquad (4.19)$$

where $L_k = \frac{1-\alpha_k}{p_k(x_k^* + \gamma_k)}$, $\omega_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k} + 1\right)^{\alpha_k - 1}$ for $k = 1, 2, ..., K$, $z_{in} = 1$ if $i = n$, and $z_{in} = 0$ if $i \neq n$. Unfortunately, there is no concise form for the determinant of the Jacobian for $M > 1$. When $M = 1$ (*i.e.*, only one alternative is chosen) for all individuals, there are no satiation effects ($\alpha_k = 1$ for all $k$), $\theta_{km} = 0 \, \forall \, k, m \; (k \neq m)$, and the Jacobian term drops out (that is, the continuous component drops out, because all expenditure is allocated to good 1). Then, the model in Equation (4.18) collapses to the standard MNL model.

In estimating the model just discussed, we should ensure $\tilde{\pi}_k > 0$ for each good $k$. This is recognized in the logarithmic transformation of $\tilde{\pi}_k$ appearing in $V_k$. These constraints can be imposed by using a constrained maximum likelihood procedure. At the same time, we also require that $\psi_k > 0$, which is ensured (as in the AS case) by writing $\psi_k = \exp(\boldsymbol{\beta}'\boldsymbol{z}_k)$. Also, since only differences in the $V_k$ from $V_1$ matters in the KKT conditions, a constant cannot be identified in the term for one of the $K$ alternatives. Similarly, individual-specific variables are introduced in the $V_k$'s for ($K$-1) alternatives, with the remaining alternative serving as the base. The parameters in the DU-RM non-AS-based MDCEV model may be estimated in a straightforward way using the maximum likelihood inference approach. However, it is difficult to motivate generalized extreme value error structures and variable-specific random coefficients in the context of the DU-RM formulation. These extensions, however, are quite natural in the context of the RU-DM decision postulate.

*4.3.2.2 Case of outside and inside goods*

For the DU-RM formulation with an outside good, the econometrics simplify considerably. One can go through the same procedure as earlier by writing the KKT conditions and introducing stochasticity corresponding to the deterministic utility expression in Equation (4.6) instead of Equation (4.4). For the outside good (say, the first alternative), we have the following: $\boldsymbol{\beta}'\mathbf{z}_1 = 0, \psi_1 = 1,$ and $p_1 = 1$. The final expression for probability in this outside good case is the same as in Equation (4.18) with the following modifications to the $V_k$ terms: $V_k = \ln(\tilde{\pi}_k) + (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k} + 1\right) - \ln p_k$ for $k > 2$, and $V_1 = (\alpha_1 - 1)\ln(x_1^* + \gamma_1)$.

The Jacobian elements in this case simplify relative to Equation (4.19), with $\theta_{1m} = 0 \;\; \forall m\,(m \neq 1)$. The elements now are given as follows:

$$J_{in} = p_{n+1}\left[-\omega_{n+1}\frac{\theta_{i+1,n+1}}{\tilde{\pi}_{i+1}}(1 - z_{in}) + L_1 + L_{n+1}z_{in}\right]. \tag{4.20}$$

4.3.3 The RU-RM non-AS utility formulation and model

*4.3.3.1 Case of only inside goods*

Consider the random utility function of Equation (4.9) for the case with no outside good. The KKT conditions are given by Equation (4.10), but we now add stochasticity originating from consumer mistakes in the optimizing process. Then, the KKT conditions take the form shown below:

$$\frac{\eta_k \exp(\varepsilon_k)}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda = 0, \text{ if } x_k^* > 0,\ k = 1,2,...,K$$
$$\frac{\eta_k \exp(\varepsilon_k)}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda < 0, \text{ if } x_k^* = 0,\ k = 1,2,...,K, \quad (4.21)$$

where $\eta_k$ is as defined earlier in Equation (4.10) (and has the error term $\xi_k$ embedded within), and the $\varepsilon_k$ terms are independent and identically (across alternatives) extreme value distributed. Let, $Var(\varepsilon_k) + Var(\xi_k) = \pi^2\sigma^2/6$ for $k = 1,2,...,K$. In the RU-RM formulation, we assume that the $\xi_k$ terms are normally distributed. This is particularly convenient when one wants to accommodate a flexible error covariance structure through a multivariate normal-distributed coefficient vector $\boldsymbol{\beta}$ and/or account for covariance in utilities across alternatives through the appropriate random multivariate specification for the $\xi_k$ terms. To develop the probability function for consumptions, let $Var(\varepsilon_k) = \mu^2\pi^2\sigma^2/6$ and $Var(\xi_k) = (1-\mu^2)\pi^2\sigma^2/6$ $(k = 1,2,...,K)$, where $\mu$ is a parameter to be estimated $(0 < \mu < 1)$. Then,

- When $\mu \to 0$, and when there is no covariance among the $\xi_k$ terms across alternatives, the RU-RM formulation approaches the RU-DM formulation of Section 4.3.1 in which the scale parameter $\sigma$ is innocuously rescaled to $(\pi/\sqrt{6})\,\sigma$, so that the variance of the error terms $\xi_k$ in the RU-DM formulation is comparable to the variance of the corresponding terms in the RU-RM formulation.
- When $\mu \to 1$, the RU-RM formulation approaches the DU-RM formulation (Section 4.3.2).

Thus, the parameter $\mu$ determines the extent of the mix of the RU-DM and DU-RM decision postulates leading up to the observed behavior of consumers. One can impose the constraint that $(0 < \mu < 1)$ through the use of a logistic transform $\mu = 1/(1+\exp(-\mu^*))$ and estimate the parameter $\mu^*$.

The probability expression for consumptions in the RU-RM model formulation takes the following mixed MDCEV form:

$$P\left(x_1^*, x_2^*, x_3^*, ..., x_M^*, 0, 0, ..., 0\right) = \int_{\xi=-\infty}^{\infty} \left[ abs(det(J|\xi) \frac{1}{(\mu\sigma)^{M-1}} \left[ \frac{\prod_{i=1}^{M} e^{[V_i/(\mu\sigma)]|\xi_i}}{\left(\sum_{k=1}^{K} e^{[V_k/(\mu\sigma)]|\xi_k}\right)^M} \right] (M-1)! \right] dF(\xi). \tag{4.22}$$

where $V_k = \ln(\eta_k) - \ln p_k + (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k} + 1\right)$, $\eta_k = \left(\psi_k \exp(\xi_k) + W_k\right)$, $W_k$ is defined as earlier, and $F$ is the multivariate normal distribution of the random element vector $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_K)$ (each of whose elements has a variance of $(1-\mu^2)(\pi^2\sigma^2)/6$). The elements of the Jacobian are given by: (a quite similar derivation is presented in Appendix D, where the deterministic term $\tilde{\pi}_k$ can be replaced by the random term $\eta_k$)

$$\begin{aligned} J_{in} \mid \boldsymbol{\xi} = \omega_{n+1} p_{n+1} &\left[ \frac{\theta_{1,n+1}}{(\eta_1 \mid \xi_1)} - (1 - z_{in}) \frac{\theta_{i+1,n+1}}{(\eta_{i+1} \mid \xi_{i+1})} \right] \\ &+ \omega_1 p_1^2 \frac{\theta_{1,i+1}}{(\eta_{i+1} \mid \xi_{i+1})} + p_{n+1}\left[L_1 + z_{in} L_{i+1}\right]. \end{aligned} \tag{4.23}$$

*4.3.3.2 Case of outside and inside goods*

When there is an outside good, the probability expression remains the same as in Equation (4.22), but with the following modifications: $V_k = \ln(\eta_k) - \ln p_k + (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k} + 1\right)$ $(k > 2)$, $V_1 = (\alpha_1 - 1)\ln(x_1^* + \gamma_1)$, $\theta_{1m} = 0$ $\forall m\,(m \neq 1)$, $W_1 = 0$, $\boldsymbol{\beta}'\mathbf{z}_1 = 0$, $\psi_1 = 1$, $p_1 = 1$, and $\eta_1 = \exp(\xi_1)$.

The Jacobian elements in this case are given as follows:

$$J_{in} \mid \boldsymbol{\xi} = \omega_{n+1}\left[ -(1 - z_{in}) \frac{\theta_{i+1,n+1}}{(\eta_{i+1} \mid \xi_{i+1})} \right] + p_{n+1}\left[L_1 + z_{in} L_{i+1}\right]. \tag{4.24}$$

## 4.4 Application to household transportation expenditures

4.4.1 Empirical context

In 2010, transportation expenses accounted for 12-15% of total household income and nearly 20% of total household expenses in the U.S. (U.S. Bureau of Labor Statistics, 2012). In fact, this is the second largest expense category after housing, representing an average expenditure of $ 7,677 per year (or, equivalently, about $650 per month). Despite these numbers, American households continue to increase their expenses in transportation, mainly because of an increase on fuel prices (either due to an increase in fuel price itself or due to an increase in gasoline taxes). The relevance of transportation expenditures is stressed by the fact that fuel demand is highly inelastic (Havranek *et al*., 2012, Brons *et al.*, 2008, Small and Van Dender, 2007), suggesting that any substantial change in fuel prices would lead to an increase in transportation expenditures. Expenses on transportation impose an evident burden to household budgets, particularly for low-income families. It is little surprise, therefore, that the study of transportation expenditures has been of much interest in recent years (Gicheva *et al.*, 2007, Cooper, 2005, Hughes *et al.*, 2006, Thakuriah and Liao, 2006, Choo *et al.*, 2007a, Sanchez *et al.*, 2006). Several of these studies examine the factors that influence total household transportation expenditures and/or examine transportation expenditures in relation to expenditures on other commodities and services (such as in relation to housing, telecommunications, groceries, and eating out). But there has been relatively little research on identifying the many disaggregate-level components of transportation expenditures – rather all transportation expenditures are usually lumped into a single category.

A number of studies have acknowledged the presence of complementarity and substitution effects in transportation-related expenditures. For example, research has been undertaken to study expenditure in competitive transportation modes (Taplin, 1980, Oum and Gillen, 1983, Andrikopoulos and Brox, 1990). However, these studies have used indirect utility approaches to estimate cross-elasticity effects. The almost ideal demand system (AIDS) proposed by Deaton and Muellbauer (1980) has been applied to study complementarity between transportation expenditures (as an aggregate category) and other expenditures categories (see, for example, Arranz, 2001, Choo *et al*., 2007b, Mokhtarian *et al*., 2011), and between transportation modes expenditures using panel data (Tsekeris, 2008). Although the AIDS is based on a unified utility maximization principle and is useful to identify possible substitution or complementarity patterns, it assumes that all expenditure categories are chosen by all households (that is, it does not allow corner solutions). To our knowledge, this is the first study to explicitly accommodate rich substitution patterns as well as allowing complementarity among disaggregate transportation expenditure categories.

### 4.4.2 Data description

Data for the analysis is drawn from the 2002 Consumer Expenditure Survey (CES), which is a national level survey conducted by the U.S. Census Bureau (U.S. Bureau of Labor Statistics, 2003). This survey has been carried out regularly since 1980 and is designed to collect information on incomes, expenditures and buying habits of consumers in the United States. In addition, information on individual and household socio-economic, demographic, employment, and vehicle characteristics is also collected.

To show the applicability of our proposed non-AS model, we used an expenditure data that Ferdous *et al.* (2010) put together from the CES survey. The dependent variable for the analysis is annual household expenditure in six disaggregate categories:

1. Vehicle purchase
2. Gasoline and motor oil (termed as gasoline in the rest of the document)
3. Vehicle insurance
4. Vehicle operation and maintenance (labeled as vehicle maintenance from hereon)
5. Air travel
6. Public transportation

Details of the data and sample extraction process for the current analysis are available in Ferdous *et al*. (2010). Essentially, the 109 categories of expenditure and income defined by the CES were consolidated, defining 17 broad categories of annual expenditure. The household budget was constructed based on the household savings (defined as expenditure minus income). Then, if the savings were positive, the budget $E$ (see Equation 4.7) is equal to the sum of expenditures; otherwise, the budget is equal to the income. Finally, out of the 17 categories defined by Ferdous *et al.*, the 11 categories not listed above were considered in a single "outside good" category that lumps all non-transportation expenditures (such as expenditure on housing, food and utilities), so that total transportation expenditure is endogenously determined.

The final sample for analysis includes 4100 households with the information identified above. Figure 4.1 shows the average annual household expenditure in the six transportation-related categories used for the analysis. The figure shows that, on average, the largest household expenditure goes to new and used vehicle purchases ($3155), followed by vehicle maintenance (about $1300), gasoline (almost $1200) and vehicle insurance ($870). Consequently, an average American household spends almost $6500 per year in vehicle-related costs. Expenditure on air travel and public transportation is low compared with the vehicle-related categories. In particular, average household expenditure on public transportation is only $107 per year. This low cost is a result not only of the affordability of public transportation, but also of its low usage.

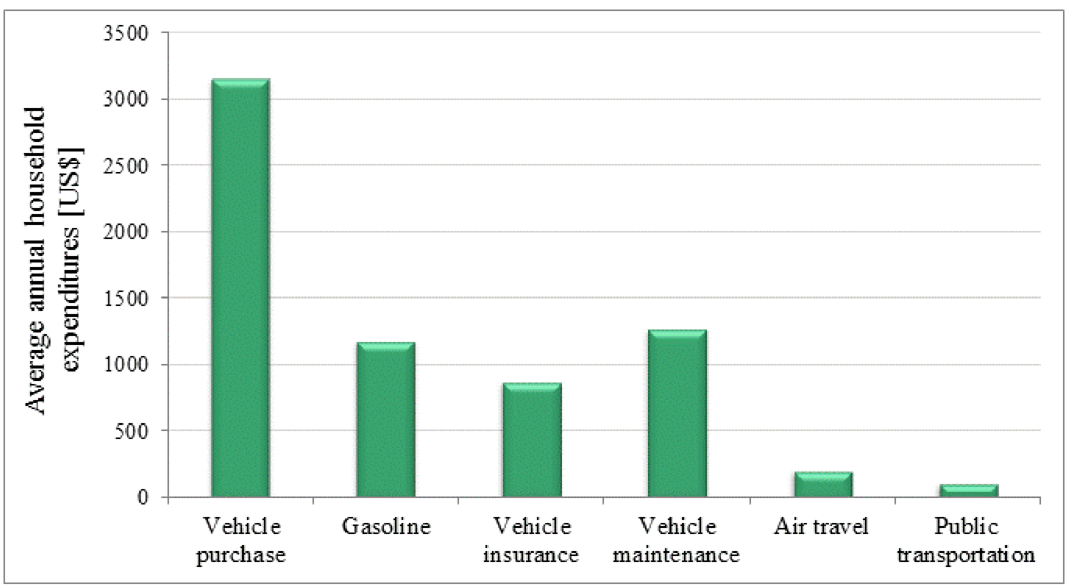


Figure 4.7: Average annual household expenditure in transportation-related categories

Since annual household income is considered exogenous in the current analysis, we use the proportion of annual household income spent in each of the six transportation categories and the "outside" non-transportation category as the dependent variables in the analysis.

Of the 4100 households (HHs) in the sample, a random sample of 3600 households was used for model estimation and the remaining sample of 500 households was withheld for validation. The descriptive statistics of the dependent variable are presented in Table 4.1, for both the estimation sample and validation sample. The table shows that households always spend some positive amount on the "outside good" category, while expenditures are zero for one or more transportation categories for some households. In other words, the outside good is also an essential good and the transportation-related categories are inside goods.

Table 4.1: Household annual expenditure statistics, estimation and validation samples

| Alternatives | Estimation sample | | | Validation sample | | |
|---|---|---|---|---|---|---|
| | **HHs spending in… [%]** | **Average Expenditure [%]** | **Expenditure Std. Dev. [%]** | **HHs spending in… [%]** | **Average Expenditure [%]** | **Expenditure Std. Dev. [%]** |
| Non-transportation related (outside good) | 100.00 | 85.30 | 14.08 | 100.00 | 84.74 | 12.01 |
| Vehicle purchase | 25.06 | 5.93 | 13.13 | 35.27 | 6.56 | 11.21 |
| Gasoline | 92.97 | 2.96 | 2.20 | 99.80 | 2.77 | 1.69 |
| Vehicle insurance | 78.91 | 2.20 | 2.07 | 91.58 | 1.93 | 1.71 |
| Vehicle maintenance | 88.80 | 2.87 | 3.24 | 99.00 | 3.04 | 2.68 |
| Air travel | 28.62 | 0.44 | 1.22 | 52.71 | 0.67 | 1.17 |
| Public transportation | 33.62 | 0.30 | 0.91 | 48.30 | 0.29 | 0.75 |
| Number of observations | | 3600 | | | 500 | |

Table 4.1 shows that the validation sample and estimation sample are very similar in terms of annual expenditure. About one-quarter of the estimation sample reports expenditures on vehicle purchase. About 93% of the estimation sample incurs expenditures on gasoline, and about 89% of the sample indicates vehicle maintenance expenses. About 79% of the estimation sample has vehicle-insurance related expenses, suggesting that a sizeable number of households operate motor vehicles with no

insurance or have insurance costs paid for them (possibly by an employer or self-employed business). About one-third of the estimation sample reports spending money on public transportation and air travel. Only 2.6% of the households expend no money in transportation-related expenses. These households may undertake trips using non-motorized modes, or rely on someone else to travel. All together, expenditures on transportation-related items account for about 15% of household income, a figure that is quite consistent with reported national figures.

### 4.4.3 Model estimation results

The AS and non-AS models were estimated using the Gauss matrix programming language. We first estimated the best empirical specification for the MDCEV model (assuming additive separability) based on intuitive and statistical significance considerations, and then explored alternative specifications for the interaction parameters in the non-AS model for the RU-DM formulation, the DU-RM formulation and the RU-RM formulation. The $\gamma$ profile of Equation (4.5) was used in all specifications, since it consistently provided better model fit than the $\alpha$ profile. Also, the $\gamma$ value for the outside good was set to zero for estimation stability. Recall that the RU-DM specification assumes normally distributed random terms for the analyst's errors in characterizing the consumer's utility functions. The DU-RM formulation assumes extreme value random error terms for the random mistakes made by the consumer during his/her optimization process. Finally, the RU-RM formulation utilizes a combination of extreme value error terms and normally distributed error terms, for the consumer's mistakes and the analyst's errors, respectively. In the absence of interactions between the sub-utility functions of different alternatives, the RU-DM formulation collapses to an AS MDC model with IID normal error terms (label this as the MDCN model), while the DU-RM formulation collapses to the MDCEV model. Thus, for model evaluation purposes, the analyst can compare the performance of the RU-DM model to its special case MDCN and that of the DU-RM model to its special case MDCEV. The RU-RM model has no direct AS model to be compared with, as its non-additive simile is a combination of the MDCEV model and the MDCN model.

For the estimation of the RU-DM non-AS model, as discussed in Section 4.3.1, two sets of conditions need to be considered. The first condition is to ensure a strictly increasing utility function (by constraining $\eta_k > 0$, from Equation (4.10)) while the second condition is to ensure that the $R_k \mid \xi_1$ terms are positive (since these terms are inside a logarithmic function). These two sets of conditions are conflicting in nature. That is, negative values of interaction parameters ($\theta_{km}$) increase the chance of violating the former constraint while positive interaction parameters increase the chance of violating the latter constraint. In the current empirical application, attempts to impose these conflicting constraints were faced with estimation instability and convergence problems. Thus, for estimation purposes, only the latter condition ($R_k \mid \xi_1 > 0, \forall k$) was considered assuming that the former condition is not violated at optimal consumptions (since the consumption point at which the marginal utility of a good becomes negative cannot be an optimal consumption point). The DU-RM non-AS model was estimated using the constrained maximum likelihood module of Gauss to explicitly consider the constraint that $\tilde{\pi}_k > 0$ for each good *k* (since the term $\tilde{\pi}_k$ is inside a logarithmic function). The RU-RM non-AS model was estimated using maximum likelihood procedures while imposing that the baseline marginal utility was positive ($\tilde{\pi}_k > 0 \ \ \forall k$). To evaluate the multivariate integral of Equation (4.23), we used the Halton sequence to draw realizations for $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_K)$ from a normal distribution, assuming that these error terms are IID. Details of the Halton sequence and the procedure to generate this sequence are available in Bhat (2003). We tested the sensitivity of parameter estimates with different numbers of Halton draws per observation, and found the results to be very stable with as few as 75 draws. In this analysis we used 100 draws per household in the estimation.

The estimation results of the baseline marginal parameters are provided in Table 4.2. The table is organized into three major columns. The first major column provides the parameters estimates assuming an IID normal distribution over the error terms. The second major column provides the estimation results under the assumption of an IID extreme value distribution over the error terms. Each of these major columns is divided into two sub-columns, presenting the estimates of the AS (MDCN and MDCEV) and non-AS (RU-DM and DU-RM) models. The third column provides the parameters estimates of the RU-RM non-AS model, which assumes both normal and extreme value error terms. In this table, if there are no coefficients corresponding to a variable for certain expenditure categories, it implies that these categories constitute the base expenditure categories off which the coefficients on that variable for other categories need to be interpreted. A positive (negative) coefficient for a certain variable-category combination means that an increase in the explanatory variable increases (decreases) the likelihood of budget being allocated to that expenditure category relative to the base expenditure categories. For example, as the number of workers per household increases, the proportion of total income share expended on vehicle purchase increases relative to other categories.

The alternative specific constants in the baseline utility for all the transportation categories are negative, indicating the generally higher baseline utility of the "outside" non-transportation good category relative to each transportation category (this is a reflection of the higher expenditure on the outside good than on the transportation categories).

The satiation parameters ($\gamma_k$) in Table 4.2 capture the variation in the extent of non-linearity across different expenditure categories and indicate statistically significant satiation. The value is highest for the vehicle purchase category, indicating that households are likely to allocate a large proportion of their budget to acquiring a vehicle, if they expend any money in this category. The lowest value is for gasoline, suggesting that the lowest proportion of money is allocated to this category and satiation is reached very quickly.

Table 4.2: Non-AS estimation results - Baseline utility and translation parameters

| Variables | Normal error terms | | | | Extreme value error term | | | | RU-RM non-AS model | |
|---|---|---|---|---|---|---|---|---|---|---|
| | MDCN model | | RU-DM non-AS model | | MDCEV model | | DU-RM non-AS model | | | |
| | **Parameter** | **t-stat** | **Parameter** | **t-stat** | **Parameter** | **t-stat** | **Parameter** | **t-stat** | **Parameter** | **t-stat** |
| **Baseline Utility Parameters ( $\beta$ )** | | | | | | | | | | |
| *Baseline constants* | | | | | | | | | | |
| Vehicle purchase | -6.414 | -88.79 | -6.577 | -97.80 | -7.126 | -70.59 | -8.143 | -18.27 | -5.926 | -106.70 |
| Gasoline | -2.922 | -45.42 | -2.584 | -34.26 | -2.523 | -37.62 | -2.919 | -33.56 | -3.453 | -100.43 |
| Vehicle insurance | -4.030 | -68.43 | -3.952 | -75.25 | -3.975 | -72.08 | -4.475 | -28.22 | -4.329 | -120.54 |
| Vehicle maintenance | -3.600 | -57.26 | -3.574 | -60.89 | -3.446 | -60.82 | -4.230 | -30.29 | -4.169 | -135.08 |
| Air travel | -5.684 | -90.73 | -5.322 | -66.26 | -6.144 | -72.87 | -5.196 | -41.96 | -5.931 | -76.82 |
| Public transportation | -5.439 | -56.90 | -3.923 | -33.46 | -5.819 | -42.16 | -4.699 | -48.43 | -5.893 | -38.35 |
| *Number of workers in household* | | | | | | | | | | |
| Vehicle purchase | 0.123 | 3.81 | 0.147 | 4.61 | 0.182 | 4.41 | 0.189 | 3.53 | 0.079 | 3.62 |
| Gasoline | 0.207 | 6.09 | 0.228 | 6.40 | 0.209 | 7.74 | 0.254 | 5.74 | 0.165 | 10.64 |
| Vehicle insurance | 0.081 | 3.07 | 0.107 | 3.91 | 0.081 | 2.89 | 0.104 | 2.54 | 0.039 | 2.30 |
| Vehicle maintenance | 0.187 | 7.58 | 0.227 | 9.20 | 0.192 | 7.36 | 0.281 | 6.14 | 0.098 | 7.71 |
| *Annual HH income $30,000 - $70,000* | | | | | | | | | | |
| Vehicle purchase | 0.611 | 9.34 | 0.713 | 10.94 | 0.808 | 7.97 | 1.404 | 4.10 | 0.513 | 10.37 |
| Gasoline | -0.215 | -2.79 | -0.148 | -1.96 | -0.284 | -5.60 | -0.300 | -3.03 | -0.219 | -7.51 |
| Air travel | 0.537 | 8.28 | 0.542 | 9.68 | 0.756 | 8.80 | 0.281 | 5.01 | 0.330 | 4.43 |
| *Annual HH income >$70,000* | | | | | | | | | | |
| Vehicle purchase | 0.579 | 5.96 | 0.757 | 8.27 | 0.805 | 6.34 | 1.461 | 4.03 | 0.509 | 7.88 |
| Gasoline | -0.730 | -5.59 | -0.592 | -4.82 | -0.793 | -10.89 | -0.881 | -5.22 | -0.636 | -13.91 |
| Vehicle insurance | -0.374 | -4.20 | -0.283 | -3.27 | -0.337 | -5.26 | -0.332 | -2.81 | -0.251 | -5.34 |
| Air travel | 0.800 | 7.57 | 0.803 | 9.89 | 1.189 | 11.31 | 0.536 | 5.44 | 0.290 | 2.80 |

Table 4.2: Non-AS estimation results - Baseline utility and translation parameters (cont.)

| Variables | Normal error terms | | | | Extreme value error term | | | | RU-RM non-AS model | |
|---|---|---|---|---|---|---|---|---|---|---|
| | MDCN model | | RU-DM non-AS model | | MDCEV model | | DU-RM non-AS model | | | |
| | Parameter | t-stat | Parameter | t-stat | Parameter | t-stat | Parameter | t-stat | Parameter | t-stat |
| **Baseline Utility Parameters ( $\beta$ )** | | | | | | | | | | |
| *Number of vehicles in household* | | | | | | | | | | |
| Vehicle purchase | 0.232 | 10.01 | 0.282 | 14.17 | 0.304 | 11.75 | 0.379 | 11.38 | 0.149 | 11.68 |
| Gasoline | 0.317 | 11.15 | 0.352 | 14.20 | 0.305 | 15.70 | 0.387 | 14.11 | 0.177 | 20.44 |
| Vehicle insurance | 0.287 | 12.71 | 0.329 | 17.49 | 0.275 | 14.04 | 0.348 | 12.03 | 0.151 | 12.76 |
| Vehicle maintenance | 0.270 | 12.99 | 0.318 | 17.61 | 0.269 | 13.62 | 0.364 | 13.65 | 0.105 | 11.96 |
| Air travel | 0.070 | 2.80 | 0.115 | 5.57 | 0.073 | 2.56 | 0.084 | 4.79 | -0.030 | -1.20 |
| Public transportation | -0.082 | -3.90 | -0.019 | -1.13 | -0.122 | -3.82 | 0.010 | 1.19 | -0.698 | -25.25 |
| *For the public transportation category only* | | | | | | | | | | |
| Non-Caucasian HH | 0.468 | 9.73 | 0.314 | 5.69 | 0.417 | 5.29 | 0.084 | 2.02 | 0.712 | 9.75 |
| Urban location | 0.355 | 4.65 | 0.293 | 3.82 | 0.490 | 3.96 | 0.085 | 2.84 | 0.487 | 3.36 |
| North East Region | 0.708 | 14.17 | 0.587 | 10.53 | 0.722 | 9.04 | 0.182 | 3.80 | 0.873 | 11.32 |
| Western Region | 0.411 | 8.41 | 0.393 | 7.59 | 0.590 | 8.28 | 0.131 | 3.48 | 0.398 | 5.59 |
| **Translation Parameters ( $\gamma_k$ )** | | | | | | | | | | |
| Vehicle purchase | 36.609 | 13.00 | 36.626 | 12.97 | 20.888 | 15.31 | 21.607 | 10.67 | 80.185 | 9.82 |
| Gasoline | 0.268 | 11.78 | 0.174 | 9.62 | 0.196 | 17.49 | 0.166 | 8.81 | 0.744 | 18.24 |
| Vehicle insurance | 0.683 | 19.82 | 0.580 | 18.32 | 0.613 | 27.13 | 0.579 | 15.78 | 1.568 | 26.30 |
| Vehicle maintenance | 0.386 | 17.65 | 0.339 | 17.23 | 0.284 | 21.08 | 0.269 | 16.12 | 1.809 | 23.95 |
| Air travel | 1.121 | 18.78 | 0.805 | 13.00 | 0.677 | 19.58 | 0.548 | 13.44 | 8.314 | 16.48 |
| Public transportation | 0.491 | 25.06 | 0.133 | 9.53 | 0.237 | 19.64 | 0.187 | 16.81 | 1.330 | 18.33 |
| **Log-likelihood at convergence** | -36,301 | | -35,921 | | -37,045 | | -36,522 | | -34,168 | |
| **Number of parameters** | 32 | | | | | | | | | |
| **Number of observations** | 3600 | | | | | | | | | |

Overall, the results in Table 4.2 are intuitive. Also, while there are differences in the estimated coefficients between the corresponding AS and non-AS models, the general pattern and direction of variable effects are similar. As the number of workers increases, so does the proportion of income allocated to all vehicle-related transportation expenses, presumably to support the transportation needs of multi-worker households (a similar result was found by Thakuriah and Liao (2005) in the context of vehicle expenses). Regarding income, the middle-income group (\$30,000 - \$70,000 annual income) spends a lower proportion of its income on gasoline relative to the low income group. This result indicates that transportation expenditures constitute a major share of expenditures for the low-income group. A detailed discussion of this result from a social and environmental justice perspective can be found in Deka (2004). Higher income groups also tend to spend a lower proportion of their resources on gasoline, most likely due to a travel saturation effect combined with high income. As one would expect, the proportion of vehicle insurance expenses decreases as income rises, while the proportion on new vehicle purchases and air travel increases as income rises. Multicar households tend to allocate a greater proportion of their income to all transportation categories, except on public transportation (the effect of multicar households on public transportation is negative in the MDCEV, MDCEN and RU-DM models, and positive but statistically insignificant in the DU-RM non-AS model). Note that the effect of number of vehicles on air travel expenditures is positive for all models but the RU-RM model; however, this parameter has low statistical significance. Finally, non-Caucasians, those residing in urban areas, and those living in the Northeast and West regions of the U.S. spend a higher proportion on public transportation than Caucasians, those residing in non-urban areas, and those living in the South and Midwest regions of the U.S., respectively.

As mentioned in Section 4.3.3, the RU-RM non-AS formulation combines two postulates of consumer behavior (RU-DM and DU-RM) via the parameter $\mu$. In the current empirical analysis, we obtained $\mu = 0.379$. The value of the parameter supports the hypothesis that household expenditure choices combine two sources of error: the parameter is statistically different from zero (with a t-stat of 58.51) and significantly different from one (with a t-stat of 95.60). The $\mu$ parameter, being closer to zero than one, indicates that the prevalent source of stochasticity is due to the analyst's errors in characterizing the consumer's utility function (RU-DM formulation), while also, to a lesser extent, stochasticity  arise from the random mistakes consumers make during utility maximization (DU-RM formulation).

Several interaction parameters are statistically significant in the final model specification, and they are presented in Table 4.3. The interaction parameters of the DU-RM non-AS model indicate a significant complementary effect in vehicle purchase and gasoline expenditures, and in vehicle purchase and vehicle maintenance expenditures. Also, as expected, there are complementary effects in the expenditures on gasoline, vehicle insurance, and vehicle maintenance, as well as between air travel and public transportation expenditures. This last complementary effect perhaps reflects the use of public transportation to get to/from the airport and the use of public transportation at the non-home end. On the other hand, there are particularly sensitive substitution effects in gasoline and air/public transportation expenditures, and vehicle insurance and air/public transportation expenditures. Such complementary and rich substitution effects are not possible within the AS utility formulation of the MDCEV model framework, and require the non-additive utility formulation of the non-AS framework proposed here. For the RU-DM non-AS model formulation, only substitution effects were statistically significant. While the interpretations of these substitution effects align with the results of the DU-RM non-AS model, it is not clear why no complementarity effects (*i.e.*, positive interaction parameters) were estimated to be statistically significant. Nevertheless, as discussed in the next section, both the non-AS formulations (*i.e.*, DU-RM and RU-DM formulations) were found to be better than their AS counterparts. The RU-RM model interaction parameters show significant complementarity effects, as well as substation effects for vehicle purchase and public transportation. In this model, the interaction between air travel and public transportation is particularly significant, in addition to the complementarity between vehicle purchase and maintenance.

Table 4.3: Non-AS estimation results - Interaction parameters

| **Interaction Parameters ( $\theta_{km}$ )** | **RU-DM non-AS model** | | **DU-RM non-AS model** | | **RU-RM non-AS model** | |
|---|---|---|---|---|---|---|
| | **Parameter** | **t-stat** | **Parameter** | **t-stat** | **Parameter** | **t-stat** |
| Veh. purchase - gasoline | - | - | $1.355 \times 10^{-3}$ | 3.22 | - | - |
| Veh. purchase - veh. insurance | - | - | - | - | $0.300 \times 10^{-4}$ | 4.36 |
| Veh. purchase - veh. maintenance | - | - | $0.323 \times 10^{-3}$ | 2.11 | $0.131 \times 10^{-3}$ | 9.98 |
| Veh. purchase - air travel | - | - | - | - | - | - |
| Veh. purchase - public transp. | - | - | - | - | $-0.890 \times 10^{-4}$ | -4.85 |
| Gasoline - veh. insurance | - | - | $2.053 \times 10^{-2}$ | 4.09 | $2.436 \times 10^{-3}$ | 10.17 |
| Gasoline - veh. maintenance | - | - | $5.167 \times 10^{-2}$ | 6.33 | $0.909 \times 10^{-3}$ | 4.95 |
| Gasoline - air travel | $-5.171 \times 10^{-3}$ | -2.83 | $-5.216 \times 10^{-3}$ | -3.95 | - | - |
| Gasoline - public transp. | $-43.350 \times 10^{-2}$ | -5.46 | $-8.205 \times 10^{-3}$ | -4.91 | - | - |
| Veh. insurance - veh. maintenance | - | - | $3.248 \times 10^{-3}$ | 2.16 | $0.366 \times 10^{-3}$ | 4.19 |
| Veh. insurance - air travel | $-2.261 \times 10^{-3}$ | -4.10 | $-1.409 \times 10^{-3}$ | -4.14 | - | - |
| Veh. insurance - public transp. | $-11.928 \times 10^{-2}$ | -5.51 | $-2.488 \times 10^{-3}$ | -5.27 | - | - |
| Veh. maintenance - air travel | - | - | - | - | - | - |
| Veh. maintenance - public transp. | $-10.722 \times 10^{-2}$ | -4.03 | - | - | - | - |
| Air travel - public transp. | - | - | $6.607\times10^{-3}$ | 10.78 | $9.204 \times 10^{-3}$ | 35.87 |
| **Number of parameters** | 5 | | 10 | | 7 | |

## 4.4.4 Model evaluation

In this section, we compare the model performance of the AS and non-AS models both in the estimation sample of 3600 households as well as a validation sample of 500 households.

In terms of model fit in the estimation sample, we compare two nested models using the likelihood ratio test. To undertake this test, stack the parameters $\boldsymbol{\gamma} = (\gamma_2, \gamma_3, ..., \gamma_K)'$ and $\boldsymbol{\theta} = (\theta_{23}, \theta_{24}, ..., \theta_{K-1,K})'$, and let $\boldsymbol{\delta} = (\boldsymbol{\beta}', \boldsymbol{\gamma}', \boldsymbol{\theta}')'$. Let $d$ be the dimension of the interaction parameter $\boldsymbol{\theta}$ ( $d = 5$ for the RU-DM model, $d = 10$ for the DU-RM model, and $d = 7$ for the RU-RM model). Consider the null hypothesis $H_0 : \boldsymbol{\theta} = 0$ against $H_1 : \boldsymbol{\theta} \neq 0$. Then, the likelihood ratio test $LR$ is given by:

$$LR = -2(L(\hat{\boldsymbol{\delta}}_0) - L(\hat{\boldsymbol{\delta}})), \tag{4.25}$$

where $L(\hat{\boldsymbol{\delta}}_0)$ is the log-likelihood at convergence under the null hypothesis and $L(\hat{\boldsymbol{\delta}})$ is the log-likelihood at convergence under the alternative hypothesis. In other words, $\hat{\boldsymbol{\delta}}_0$ is the estimator of the restricted model (AS) and $\hat{\boldsymbol{\delta}}$ is the estimator of the unrestricted model (non-AS). The $LR$ statistic follows a chi-square asymptotic distribution with $d$ degrees of freedom.

- The log-likelihood value at convergence of the RU-DM non-AS model is -35,921, while that of the MDCN model is -36,301. The $LR$ test between these two models returns a value of 760, which is larger than the chi-squared statistic value with 5 degrees of freedom at any reasonable level of significance, indicating the substantially superior fit of the RU-DM non-AS model compared to the MDCN model.
- Similar results are found when comparing the MDCEV and DU-RM models. The log-likelihood value at convergence of the DU-RM non-AS model is -36,522, while that of the MDCEV model is -37,045. The corresponding likelihood ratio test is 1,046, implying that the DU-RM non-AS model is statistically superior to the MDCEV model.
- The log-likelihood value at convergence of the RU-RM non-AS model is -34,168, which is considerable higher that the same figure for the MDCEV and MDCN. Because the error terms of the RU-RM model are both normal and extreme value distributed, the test $LR$ cannot be used to compare the improvement in statistical fit.

Of course, both the AS and non-AS models at convergence provide a much better data fit than the naïve AS model with only the alternative-specific constant terms and the translation parameters (with the effect of all explanatory variables assumed to be zero), which has a log-likelihood value of -37,692 for the MDCEV and -37,185 for the MDCN.

To further compare the performance of the AS and non-AS models, we computed an out-of-sample log-likelihood function (OSLLF) using the validation sample of 500 observations for the AS model with independent variables, and the non-AS model. The OSLLF is computed by plugging in the out-of-sample (*i.e.*, validation) observations into the log-likelihood function, while retaining the estimated parameters from the estimation sample. As indicated by Norwood *et al.* (2001), the model with the highest value of OSLLF is the preferred one, since it is most likely to generate the set of out-of-sample observations. Table 4.4 reports the OSLLF values for the entire validation sample (of 500 households) as well as for different socio-demographic segments within the sample. As can be observed from the first row, the OSLLF for the non-AS model is higher than the AS MDC models, for all the RU-DM, DU-RM and RU-RM formulations. Further, the OSLLF for the non-AS models is, in general, higher than the OSLLF for the AS models for all socio-demographic segments, except for the some segments of "number of vehicles" and "race".

In summary, the data fit of the non-AS models is superior to that of the AS models in both estimation and validation samples.

Table 4.4: Out-of-sample log-likelihood function (OSLLF) in the validation sample

| Sample details | Number of observations | Normal Errors Terms | | Extreme Value Error Terms | | RU-RM non-AS model |
|---|---|---|---|---|---|---|
| | | MDCN model | RU-DM non-AS model | MDCEV model | DU-RM non-AS model | |
| Full validation sample | 500 | -5449.68 | -5413.20 | -5575.23 | -5475.03 | -5179.57 |
| *Number of workers in HH* | | | | | | |
| 0 | 14 | -144.73 | -144.28 | -147.99 | -145.57 | -136.35 |
| 1 | 109 | -1109.94 | -1098.22 | -1139.69 | -1119.53 | -1059.11 |
| 2 | 240 | -2612.93 | -2589.79 | -2667.62 | -2608.45 | -2433.19 |
| >2 | 137 | -1582.25 | -1580.92 | -1619.94 | -1601.48 | -1515.07 |
| *Household income ($/year)* | | | | | | |
| < 30,000 | 10 | -98.80 | -96.90 | -100.62 | -100.40 | -100.27 |
| 30,000 - 70,000 | 168 | -1807.80 | -1798.42 | -1862.08 | -1835.87 | -1702.33 |
| > 70,000 | 322 | -3543.23 | -3517.87 | -3612.53 | -3538.75 | -3362.76 |
| *Number of vehicles* | | | | | | |
| 0 | 9 | -95.83 | -90.79 | -98.68 | -140.18 | -100.06 |
| 1 | 81 | -829.86 | -836.14 | -854.90 | -1004.42 | -783.27 |
| 2 | 173 | -1723.49 | -1705.71 | -1763.61 | -1819.24 | -1690.87 |
| More than 2 | 237 | -2800.49 | -2780.58 | -2858.05 | -2511.19 | -2571.36 |
| *Race* | | | | | | |
| Non-Caucasian | 47 | -509.49 | -511.52 | -527.42 | -4953.62 | -494.47 |
| Caucasian | 453 | -4940.18 | -4901.67 | -5047.80 | -521.41 | -4630.92 |
| *Residential location* | | | | | | |
| Urban | 469 | -5099.11 | -5062.01 | -5217.53 | -5125.17 | -4855.93 |
| Rural | 31 | -350.68 | -351.17 | -357.72 | -349.86 | -321.51 |

### 4.5 Conclusions

Classical discrete and discrete-continuous models deal with situations where only one alternative is chosen from a set of mutually exclusive alternatives. Such models assume that the alternatives are perfectly substitutable for each other. On the other hand, many consumer choice situations are characterized by the simultaneous demand for multiple alternatives that are imperfect substitutes or even complements for one another. The traditional MDCEV model adopts an AS utility form that assumes that the marginal utility of a good is independent of the consumption amounts of other goods. It also is not able to allow complementarity among goods. This research develops a closed-form model formulation that allows a non-additive utility structure and complementarity effects. As importantly, the utility functional form proposed here remains within the class of flexible forms, while also retaining global theoretical consistency properties (unlike the translog and related flexible quadratic functional forms). The result is also clarity in the interpretation of the model parameters.

Stochasticity is introduced in the formulation in three different ways to develop three possible models for non-additive utility structures. In the first stochastic formulation, consumers are assumed to know all relevant factors impacting their choices and make an error-free maximization of overall utility, but the analyst is not aware of all the factors influencing consumer's choice. This is called the random utility-deterministic maximization or RU-DM decision postulate. In the second stochastic formulation, consumers are assumed to make random mistakes in maximizing utility. This is called the deterministic utility-random maximization or DU-RM decision postulate. The third stochastic formulation combines the two postulates into a random utility-random maximization or RU-RM decision postulate. The formulations of these three paradigms are quite different, and analysts can choose the most appropriate one depending on the empirical exercise under investigation.

The proposed non-AS model formulations should have several applications. In the current dissertation, we demonstrate the application of the formulations to the empirical case of household transportation expenditures in six disaggregate categories: vehicle purchase, gasoline and motor oil, vehicle insurance, vehicle operation and maintenance, air travel, and public transportation. In addition, we consider other household expenditures in a single “outside good” category that lumps all non-transportation expenditures, so that total transportation expenditure is endogenously determined. Households expend some positive amount on the “outside good” category, while expenditures are zero for one or more transportation categories for some households. Data for the analysis is drawn from the 2002 Consumer Expenditure Survey (CES), which is a national level survey conducted by the U.S. Bureau of Labor Statistics. The results of the RU-DM, DU-RM and RU-RM non-AS formulations suggest statistically significant complementary and substitution effects in the utilities of selected pairs of transportation categories, and show the substantially superior data fit of the proposed model relative to the model that assumes a separable additive utility structure. The proposed non-additive separable models performed better in a validation sample as well.

The study has successfully formulated different forms of MDC models with non-additively separable utility functional forms. But we would be remiss if we did not acknowledge the challenges we faced during the estimation of some of the proposed formulations. Future research should explore appropriate estimation procedures for the proposed formulations, especially the RU-DM and the RU-RM formulations.

# CHAPTER 5: Conclusions and Directions for Future Research

## 5.1 Multiple discrete-continuous choice models

Consumers often encounter two inter-related decisions at a choice instance – which alternative(s) to choose for consumption from a set of available alternatives, and the amount to consume of the chosen alternatives. Classical discrete choice models, such as the multinomial logit and multinomial probit, allow an analysis of consumer preferences in situations when only one alternative can be chosen for consumption from among a set of available and mutually exclusive alternatives. These models assume that the alternatives are perfect substitutes for one another. However, there are several multiple discrete-continuous (MDC) choice situations where consumers choose to consume multiple alternatives at the same time, along with the continuous dimension of the amount of consumption. Examples of such MDC contexts include, but are not limited to, household vehicle type holdings and usage, airline fleet mix and usage, individuals' choice of recreational destination locations and number of trips to the selected locations, activity type choice and duration spent in different activity types, brand choice and purchase quantity, energy equipment choice and energy consumption, and stock selection and investment amount.

In the past few decades, the modeling of MDC choices has attracted significant attention in many disciplines. A variety of modeling approaches have been used in the literature to accommodate MDC choice contexts. Among these approaches, the use of an explicit utility-maximizing framework for multiple discreteness has gained traction due to its close tie to an underlying behavioral theory. The utility maximization framework assumes a non-linear utility structure to accommodate decreasing marginal utility (or satiation) with increasing consumption. Consumers are assumed to maximize this utility subject to a budget constraint. The optimal consumption quantities (including possibly zero consumptions of some alternatives) are obtained by writing the Karush-Kuhn-Tucker (KKT) first-order conditions of the utility function with respect to the consumption quantities. Researchers from many disciplines have used such a KKT approach, and several additively separable and non-linear utility structures have been proposed in the literature. Among the available modeling frameworks, the recently developed MDCEV model structure proposed by Bhat (2005, 2008) is particularly attractive because it offers a simple closed-form consumption probability expression and employs a utility specification that enables a clear interpretation of the utility parameters and a convenient specification of the alternative attributes. Further, the MDCEV general utility form subsumes other non-linear utility forms as special cases. Stochasticity is introduced in the baseline preference for each alternative to acknowledge the presence of unobserved (to the analyst) factors that may impact the utility of each alternative (the baseline preference is the marginal utility of each alternative at the point of zero consumption of the alternative). Since the baseline preference has to be positive for the overall utility function to be valid, the baseline preference is parameterized as the exponential of a systematic component (capturing the effect of exogenous variables) as well as a stochastic error term. Assuming that the stochastic error term is identically and independently distributed (IID) extreme value, a closed probability function is obtained for the MDCEV model.

In the recent past, there has been increasing recognition of the need to extend the basic formulation of the MDCEV model. Almost all studies that have extended the MDCEV model have focused on relaxing the IID assumption made over the stochastic error term. Although relaxing this assumption can potentially improve the model data fit by accommodating richer patterns of heterogeneity in consumer preferences and allowing flexibility in distributional assumptions, other important issues such as latent choice set generation, multiple resource constraints, and the relaxation of the additively separable utility function form have received scant attention. The research in this dissertation contributes to fill these gaps in the literature.

## 5.2 Dissertation contributions

The dissertation has two main objectives. The first is to advance the formulation and the econometric modeling of MDC choice situations. The second is to contribute to the transportation literature by estimating MDC models that provide new insights on individuals' travel decision processes. As part of the first objective, this dissertation research addresses and accommodates a latent choice set generation process, multiple constraints in the resources available for consumption, and a non-additively separable utility structure. As part of the second objective, the dissertation research has examined the activity generation and time-use decisions of individuals, as well as workers' scheduling of activity participations and travel mileage in different time periods of the day. Specific contributions of the dissertation include the following.

### 5.2.1 Latent choice set generation framework

A conceptual framework to incorporate a latent choice set generation model within the MDCEV was proposed and estimated in Chapter 2. This framework recognizes that decision-makers do not necessarily consider all the available alternatives when making a choice and that the consideration choice set is not observed by the analyst. Moreover, the incorporation of varying choice sets across individuals allows the accommodation of non-compensatory behavior in the choice process. The framework was applied to model workers' participation in and travel mileage allocated to non-work activities during various time periods of the day. Five time-of-day blocks are defined for workers based on the period of the day in relation to the work schedule. The two-component model system was applied to a survey sample drawn from the San Francisco area of the United States, using data from the 2009 National Household Travel Survey (NHTS). Variables used in the analysis include a wide range of individual, household, work-related, mobility and situation, and household location characteristics. The proposed model was shown to perform substantially better than a MDCEV model that assumes a constant choice set across the sample. More importantly, the results show that the consideration set for each individual is different and dependent on the explanatory variables used for the analysis.

5.2.2 Multiple constraints framework

In Chapter 3, the MDCEV model was extended to accommodate multiple linear constraints. In numerous empirical contexts, multiple types of resources, such as time, money and space, are required to consume goods. However, most MDC studies ignore the effect that constraints may have on consumption. The multiple constrained MDCEV (MC-MDCEV) proposed in this dissertation relaxes the unrealistic assumption that individuals face only one resource constraint when selecting alternatives. The proposed formulation explicitly acknowledges the existence of multiple constraints, using a flexible and general utility function form that is applicable to the case of complete demand systems as well as incomplete demand systems. Several identification considerations necessary for parameter estimation are also discussed. The MC-MDCEV model was applied to time-use decisions, where individuals were assumed to maximize their utility from time-use in one or more activities subject to monetary and time availability constraints. The sample for the empirical exercise was generated by combining time-use information from the 2008 American Time Use Survey (ATUS) and expenditure records from the 2008 U.S. Consumer Expenditure Survey (CES). The estimation results showed that preferences can get severely mis-estimated, and the data fit can degrade substantially, when only a subset of active resource constraints is used.

5.2.3 Non-additively separable utility function

Model formulations for MDC choices were developed to allow a non-additive utility structure in Chapter 4. The formulations allow accommodating rich substitution structures and complementarity effects. As importantly, the proposed utility functional form remains within the class of flexible forms, while also retaining global theoretical consistency properties. The result is also clarity in the interpretation of the model parameters. Three different non-additive utility formulations are proposed based on alternative specifications and interpretations of stochasticity: (1) the random utility deterministic maximization (RU-DM) formulation, which considers stochasticity due to the analyst's errors in characterizing the consumer's utility function, (2) the deterministic utility random maximization (DU-RM) formulation, which considers stochasticity due to the random mistakes consumers make during utility maximization, and (3) the random utility random maximization (RU-RM) formulation, which considers both analyst's errors and consumer's mistakes within a unified framework. When applied to the consumer expenditure survey data in the United States, the non-additively separable MDC models (RU-DM, DU-RM and RU-RM) perform better than the additively separable counterparts, and suggest the presence of substitution and complementarity patterns in consumption.

In closing, the model structures proposed in this dissertation offer rigorous approaches for incorporating more flexible assumptions regarding consumer behavior. The models developed in this research, to our knowledge, represent the first formulations and applications of such approaches for accommodating a latent choice set generation, multiple constraints and non-additively separable utility structures in the context of the MDCEV model. Since the approaches are applicable to a variety of MDC choice situations and accommodate to complete and incomplete demand systems, they should be very appealing for application in several other modeling contexts.

### 5.2 Limitations of the current research and directions for future work

This dissertation makes several research contributions, as discussed in the previous section. However, there are, of course, limitations of the current research work that need to be explored in the future. In addition, there are research areas that may not necessarily fall under the category of limitations of the current research effort, but may be viewed as expanding the scope of the current work. A few of these research ideas/thoughts are discussed below.

1. The first extension of the MDCEV model develops a two-stage model to incorporate a latent choice set generation. We provide a framework for jointly modeling workers' participation in and miles of travel for non-work travel in time-of-day blocks or periods that can be defined in relation to the work schedule. Only these two dimensions of non-work activity participation are considered due to their natural importance from a travel demand management perspective, and

also to keep the model system computationally tractable while also modeling all the non-work participations jointly. From a practical implementation standpoint, further downstream models of activity scheduling in time and space (including activity purpose of non-work stops, time-of-day, chaining, and location choice) will need to be estimated to be consistent with the predictions provided by the higher-level joint participation-mileage model presented in this dissertation.

In addition, the research in this study may be extended to include detailed built environment variable effects once spatial attributes of activity locations become available in the NHTS data.

2. From a methodological standpoint, the MC-MDCEV model cannot be directly applied for activities that generate resources, because a negative unitary price (for either money or time) generates inflexions in the plane generated by the constraints, which can lead to stability problems and multiple solutions. Therefore, work activities cannot be accommodated by our model formulation. An alternative approach to include work time under our framework is to conduct a two-step budgeting. In the first step, the individual choose between work time and leisure time, given his/her wage. In the second stage the individual choose among different leisure activities, conditional on the first step. The underlying assumption of this approach is that leisure is chosen conditional upon, not jointly with, work activity.

   Another direction for future work within the MC-MDCEV modeling framework is to incorporate non-linear constraints. Linear constraints may not represent the complexity of situations consumers face in reality. For example, in some choice situations, prices vary with the amount of consumption leading to non-linear budget constraints (such as block pricing) or fixed cost may arise for some alternatives. Because the KKT conditions are not sufficient for optimality in this case, incorporating non-linear constraints in the MDCEV requires the adoption of alternative maximization approaches. Parizat and Shachar (2010) developed an MDC model with non-linear constraints, using a simulated annealing algorithm after partitioning the solution space into regions. However, as the authors acknowledged, the estimation procedure was a substantial challenge. Further studies exploring incorporating non-linear constraints may enhance our ability to represent consumer behavior.

3. The MC-MDCEV model was applied to time-use decisions with time and income constraints, and the results showed the relevance of including both constraints in terms of data fit and parameter estimation. The data used for the analysis is generated by merging time-use data records from the 2008 ATUS with expenditure records from the 2008 U.S. CES. While one can certainly debate the merits and appropriateness of such a synthetic data generation procedure, suffice it to say that the authors were not able to obtain any data set which collected both time-use and expenditure data. Given the importance of this issue in terms of the substantial benefits to be accrued from including time-use and expenditure constraints, it is hoped that concerted efforts will be undertaken in the future to obtain data on both these important drivers characterizing activity participation and time-use. In the meantime, assembling synthetic data to study the issue is the best and only possible way to proceed. Further, the imputation methods used are consistent with approaches used in a variety of fields for data imputation in which missing fields are filled by borrowing information from another record with similar attributes. Of course, in interpreting model results from any synthetic data generation procedure, an added layer of caution needs to be exercised.

4. Despite the MC-MDCEV model advantages discussed in Chapter 3, currently there is no algorithm to use the model to forecast. Forecasting is considered one of the main goals of modeling because it allows researchers to evaluate policies and to perform welfare analysis, which is particularly important in transportation planning. One approach to do so would be to use an iterative gradient-based algorithm to solve the constrained non-linear optimization problem, but this would be inefficient. Pinjari and Bhat (2012) have recently devised an algorithm

for the single constraint MDCEV case that solves the problem by building on simple, yet insightful, analytic explorations with the Karush-Kuhn-Tucker (KKT) conditions of optimality. This approach may be modified for use with the MC-MDCEV model by using the approach in an iterative fashion by cycling among the multiple constraints, while applying the approach for each constraint. Efficient cycling mechanisms should be possible. Other approaches that exploit special properties of the KKT conditions for the MC-MDCEV model can also being explored.

5. Finally, the non-additively separable (non-AS) utility structure assumes symmetry in the interaction parameters (in the notation of Chapter 4, it is assumed that $\theta_{km} = \theta_{mk}$ for all *k* and *m*). This assumption may hold for certain choice scenarios, but can be restrictive because it may be important to recognize asymmetric dependencies in consumption. Asymmetric complements refer to goods where one is more dependent on the other, yet consumers receive enhanced benefit from consuming both. In this context, Lee *et al*. (2010) developed a MDC model with asymmetric complements. Their model follows a direct utility approach and is applied to scanner panel data of cereal and milk purchases. However, Lee *at al.*'s model is developed to accommodate only two goods and does not allow substitution effects. Despite its limitations, this study is a good star point for future research in the area.

# Appendixes

## Appendix A: MDCEV model estimation results for non-work activity participation and mileage

| Explanatory Variables | Before Work | Home to Work | Work Break | Work to Home | After Work | Non-auto |
|---|---|---|---|---|---|---|
| *Individual demographics* | | | | | | |
| Age | -0.0411 (-3.41) | | | | | |
| Age 18 to 30 years | | | -0.0352 (-0.08) | | 0.2351 (0.77) | |
| Female | | | | 0.9845 (8.15) | | |
| Asian | | | -0.5155 (-1.61) | | | |
| Asian male | | 0.1428 (0.58) | | | | |
| Hispanic male | | -1.0152 (-1.98) | | -0.5283 (-1.51) | | |
| Without a driver license | | | | | | 10.6058 (17.34) |
| *Household socio-demographics* | | | | | | |
| Number of persons | | | | | 0.1962 (2.05) | |
| Number of adults | -0.7165 (-1.63) | | | -0.3810 (-2.74) | -0.2671 (-1.80) | |
| Number of children | | -0.7618 (-4.07) | | | | |
| Presence of very young children | | | | | -0.3859 (-1.16) | -0.4455 (-3.23) |
| Presence of young children | | | 0.127 (0.61) | 0.3922 (2.68) | | |
| Presence of old children | | | | | 0.7077 (2.56) | |
| Female with very young children | -1.1824 (-1.82) | 0.0019 (0.01) | -0.7324 (-1.23) | | -1.0740 (-1.89) | |
| More than one worker | | 0.6441 (3.35) | 0.5786 (2.77) | | | |
| Vehicle Availability | 0.8366 (5.96) | | | | | |
| Number of drivers | 0.7086 (1.59) | | | | | |
| Low annual household income | | | | | | 2.1499 (12.18) |
| Medium annual household income | -0.8835 (-2.61) | | | | | |
| Housing unit is owned | | | | 0.3932 (2.16) | | |

*Note: Figures in parentheses are t-statistics.*

**MDCEV model estimation results for non-work activity participation and mileage (continuation)**

| Explanatory Variables | Before Work | Home to Work | Work Break | Work to Home | After Work | Non-auto |
|---|---|---|---|---|---|---|
| *Work-related characteristics* | | | | | | |
| Flexible start time | | | | 0.4802 (4.04) | -0.1382 (-0.86) | |
| Have more than one job | | 0.3436 (1.38) | | 0.2722 (1.26) | | |
| Self-employed | 1.0186 (3.22) | 0.7310 (3.62) | 1.2956 (5.74) | | | |
| Part Time Job | 1.0468 (3.37) | | -1.4887 (-3.06) | | | |
| Distance to work < 2 miles | 1.2348 (3.97) | | | | 0.7217 (3.45) | |
| *Mobility and situational characteristics* | | | | | | |
| Number of bike trips in past week | | | | | | 0.3400 (13.48) |
| Number of walk trips in past week | | | | | | 0.3256 (38.08) |
| Trip was made alone | -8.8529 (-30.68) | -8.5464 (-42.97) | -8.0204 (-40.54) | -7.7213 (-53.03) | -9.1013 (-49.53) | |
| Monday | 0.6958 (2.45) | | | | -0.3292 (-2.26) | |
| Friday | | | -0.8618 (-2.02) | | -1.1619 (-3.08) | |
| *Household location variables* | | | | | | |
| Not in urban area | | | | | -0.4272 (-0.84) | |
| Urban size < 1 million | | | -0.3114 (-1.43) | | 0.6641 (4.04) | |
| Urban size < 1 million with access to subway or rail | -0.1959 (-0.72) | | -1.1145 (-4.89) | -0.7203 (-5.65) | | |
| *Baseline preference constants* | 0.4901 (0.55) | 1.5784 (4.88) | 0.4101 (1.62) | 0.1671 (0.69) | 0.6906 (2.13) | |
| *Satiation Parameters ($\gamma$)* | 3.7252 (8.63) | 1.2389 (8.20) | 3.3076 (10.61) | 2.1345 (15.49) | 3.9347 (10.99) | |

*Note: Figures in parentheses are t-statistics.*

## Appendix B: Computation of the determinant of the Jacobian for the multiple constrained MDCEV model

In this Appendix, for ease in presentation, we will not explicitly indicate that the Jacobian computation is conditional on the error terms $\varepsilon_1$ and $\varepsilon_2$. The elements of the Jacobian are given by:

$$J_{in} = \frac{\partial \varepsilon_{i+2}}{\partial x_{n+2}}, \quad i, n = 1,2,..., M-2, \tag{B.1}$$

where the error term of alternative $i+2$ is:

$$\varepsilon_{i+2} = \ln\left((1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}\right) - \ln \tilde{V}_{i+2} - \beta z_{i+2}, \quad i = 1,2,..., M-2 \tag{B.2}$$

Then, the $in^{th}$ element of the Jacobian is:

$$J_{in} = \frac{(1-\omega_{i+2})e^{\beta' z_1+\varepsilon_1}\dfrac{\partial \tilde{V}_1}{\partial x_{n+2}} + \omega_{i+2}e^{\beta' z_2+\varepsilon_2}\dfrac{\partial \tilde{V}_2}{\partial x_{n+2}}}{(1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}} - \frac{1}{\tilde{V}_{i+2}}\frac{\partial \tilde{V}_{i+2}}{\partial x_{n+2}}. \tag{B.3}$$

Given that $\tilde{V}_1 = \dfrac{1}{p_1}\left(\dfrac{1}{\gamma_1}\left(E - \sum_{r\neq 1}^{K} p_r x_r^*\right)+1\right)^{\alpha_1 - 1}$ and $\tilde{V}_2 = \dfrac{1}{p_2}\left(\dfrac{1}{\gamma_2}\left(E - \sum_{r\neq 2}^{K} p_r x_r^*\right)+1\right)^{\alpha_2 - 1}$,

Equation (B.3) is equivalent to:

$$J_{in} = p_{n+2}\frac{(1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1}c_1 + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}c_2}{(1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}} + \frac{1}{\tilde{V}_{i+2}}\left(\tilde{V}_{i+2}\delta_{in}c_{i+2}\right) \tag{B.4}$$

where $c_m = \dfrac{1-\alpha_m}{x_m^* + \gamma_m}$, $m = 1,2,..., M$ (all chosen alternatives), $\delta_{in} = 1$ if $i = n$ and $\delta_{in} = 0$ if $i \neq n$. Finally, the elements of the Jacobian are given by:

$$J_{in} = p_{n+2}b_{i+2} + \delta_{in}c_{i+2}, \quad i, n = 1,2,..., M-2;$$
$$b_{i+2} = \frac{(1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1}c_1 + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}c_2}{(1-\omega_{i+2})\tilde{V}_1 e^{\beta' z_1+\varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta' z_2+\varepsilon_2}} \tag{B.5}$$

To compute the determinant of the Jacobian, consider the case where the individual chooses 5 alternatives. In this case the Jacobian is the 3×3 matrix presented below.

$$J = \begin{bmatrix} b_3 p_3 + c_3 & b_3 p_4 & b_3 p_5 \\ b_4 p_3 & b_4 p_4 + c_4 & b_4 p_5 \\ b_5 p_3 & b_5 p_4 & b_5 p_5 + c_5 \end{bmatrix} \tag{B.6}$$

Because of the special structure of the Jacobian, conditional on $\varepsilon_1$ and $\varepsilon_2$, it is straight forward to see that its determinant is given by:

$$det(J) = \left( \prod_{i=1}^{3} c_{i+2} \right) \left( 1 + \sum_{i=1}^{3} \frac{p_{i+2} b_{i+2}}{c_{i+2}} \right),$$

or equivalently, $det(J) = \left( \prod_{m=3}^{5} c_m \right) \left( 1 + \sum_{m=3}^{5} \frac{p_m b_m}{c_m} \right)$ (B.7)

In the more general case with $M$ consumed alternatives, the Jacobian, after explicitly recognizing the conditionality on the error terms $\varepsilon_1$ and $\varepsilon_2$, takes the form in Equation (3.12) of Chapter 3.

## Appendix C: Computation of the determinant of the Jacobian for the random utility-deterministic maximization (RU-DM) non-additively separable utility model

For ease in presentation, we will not explicitly indicate that the Jacobian computation is conditional on the error term $\xi_1$. The elements of the Jacobian are given by:

$$J_{in} = \frac{\partial \xi_{i+1}}{\partial x_{n+1}}, \quad i,n = 1,2,...,M-1, \tag{C.1}$$

where the error term of alternative $i+1$ is:

$$\xi_{i+1} = \ln R_{i+1} - \boldsymbol{\beta}' \boldsymbol{z}_{i+1} = \ln\left(\eta_1 \frac{\omega_1}{\omega_{i+1}} - W_{i+1}\right) - \boldsymbol{\beta}'\mathbf{z}_{i+1}, \quad i = 1,2,...,M-1. \tag{C.2}$$

Then, the $in^{th}$ element of the Jacobian is:

$$J_{in} = \frac{\partial \ln R_{i+1}}{\partial x_{n+1}} = \frac{1}{R_{i+1}} \frac{\partial R_{i+1}}{\partial x_{n+1}} = \frac{1}{R_{i+1}} \frac{\partial}{\partial x_{n+1}}\left[\eta_1 \frac{\omega_1}{\omega_{i+1}}\right] - \frac{1}{R_{i+1}} \frac{\partial W_{i+1}}{\partial x_{n+1}}. \tag{C.3}$$

The first term of $J_{in}$ is given by:

$$\begin{aligned}
&\frac{1}{R_{i+1}} \frac{\partial}{\partial x_{n+1}}\left[\eta_1 \frac{\omega_1}{\omega_{i+1}}\right] = \frac{1}{R_{i+1}}\left[\frac{\omega_1}{\omega_{i+1}} \frac{\partial \eta_1}{\partial x_{n+1}} + \frac{\eta_1}{\omega_{i+1}} \frac{\partial \omega_1}{\partial x_{n+1}} + \eta_1 \omega_1 \frac{\partial (1/\omega_{i+1})}{\partial x_{n+1}} z_{in}\right] \\
&\quad = \frac{1}{R_{i+1}}\left[\frac{\omega_1}{\omega_{i+1}} \frac{\partial W_1}{\partial x_{n+1}} + \frac{\eta_1}{\omega_{i+1}} \frac{\partial \omega_1}{\partial x_{n+1}} - \frac{\eta_1 \omega_1}{\omega_{i+1}^2} \frac{\partial \omega_{i+1}}{\partial x_{n+1}} z_{in}\right] \\
&\quad = \frac{1}{R_{i+1}\omega_{i+1}}\left[\omega_1 \frac{\partial}{\partial x_{n+1}}\left\{\sum_{m \neq 1} \theta_{1m} \frac{\gamma_m}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m} - 1\right]\right\}\right. \\
&\qquad \left. + \eta_1 \frac{\partial}{\partial x_{n+1}}\left\{\frac{1}{p_1}\left(\frac{x_1^*}{\gamma_1}+1\right)^{\alpha_1 - 1}\right\} - \frac{\eta_1 \omega_1}{\omega_{i+1}} \frac{\partial}{\partial x_{n+1}}\left\{\frac{1}{p_{i+1}}\left(\frac{x_{i+1}}{\gamma_{i+1}}+1\right)^{\alpha_{i+1}-1}\right\} z_{in}\right] \\
&\quad = \frac{1}{R_{i+1}\omega_{i+1}}\left[\omega_1 \theta_{1n+1}\left(\frac{x_{n+1}}{\gamma_{n+1}}+1\right)^{\alpha_{n+1}-1} + \eta_1 \frac{1}{p_1} \frac{\partial}{\partial x_{n+1}}\left\{\left(\frac{x_1^*}{\gamma_1}+1\right)^{\alpha_1 - 1}\right\}\right. \\
&\qquad \left. - \frac{\eta_1 \omega_1}{\omega_{i+1}} \frac{1}{p_{i+1}} \frac{\partial}{\partial x_{n+1}}\left\{\left(\frac{x_{i+1}}{\gamma_{i+1}}+1\right)^{\alpha_{i+1}-1}\right\} z_{in}\right]
\end{aligned} \tag{C.4}$$

Now, define $\omega_k = \frac{1}{p_k}\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k - 1}$ and $L_k = \frac{1-\alpha_k}{p_k(x_k^* + \gamma_k)}$ for $k = 1, 2, ..., K$. Given that $x_1 = E - \sum_{r=2}^{K} p_r x_r$ , Equation (C.4) is equivalent to:

$$
\begin{aligned}
\frac{1}{R_{i+1}}\frac{\partial}{\partial x_{n+1}}\left[\eta_1 \frac{\omega_1}{\omega_{i+1}}\right] &= \frac{1}{R_{i+1}\omega_{i+1}}\left[\omega_1\theta_{1,n+1}\left(\frac{x_{n+1}}{\gamma_{n+1}}+1\right)^{\alpha_{n+1}-1} + \eta_1\omega_1\left(\frac{x_1^*}{\gamma_1}+1\right)^{-1}\frac{(1-\alpha_1)}{\gamma_1}\right. \\
&\quad \left. - \frac{\eta_1\omega_1}{\omega_{i+1}}\frac{1}{p_{i+1}}\omega_{i+1}\left(\frac{x_{i+1}}{\gamma_{i+1}}+1\right)^{-1}\frac{(\alpha_{i+1}-1)}{\gamma_{i+1}}z_{in}\right] \qquad \text{(C.5)} \\
&= \frac{\omega_1}{R_{i+1}\omega_{i+1}}\left[\theta_{1,n+1}p_{n+1}\omega_{n+1} + \eta_1 p_1^2 L_1 + \eta_1 p_{i+1}L_{i+1}z_{in}\right] \\
&= \frac{\omega_1}{R_{i+1}\omega_{i+1}}\left[\eta_1(p_1^2 L_1 + p_{i+1}L_{i+1}z_{ih}) + \theta_{1,n+1}p_{n+1}\omega_{n+1}\right]
\end{aligned}
$$

The second term of $J_{in}$ (Equation C.3) is given by:

$$
\begin{aligned}
-\frac{1}{R_{i+1}}\frac{\partial W_{i+1}}{\partial x_{n+1}} &= -\frac{1}{R_{i+1}}\frac{\partial}{\partial x_{n+1}}\left[\sum_{m\neq i+1}\theta_{i+1,m}\frac{\gamma_m}{\alpha_m}\left(\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right)\right] \\
&= -\frac{1}{R_{i+1}}\left[\theta_{1,n+1}\left(\frac{x_1}{\gamma_1}+1\right)^{\alpha_1-1}(-p_{n+1}) + \theta_{i+1,n+1}\left(\frac{x_{n+1}}{\gamma_{n+1}}+1\right)^{\alpha_{n+1}-1}(1-z_{in})\right] \qquad \text{(C.6)} \\
&= -\frac{1}{R_{i+1}}\left[-\theta_{1,n+1}p_1\omega_1 p_{n+1} + \theta_{i+1,n+1}p_{n+1}\omega_{n+1}(1-z_{in})\right] \\
&= \frac{p_{n+1}}{R_{i+1}}\left[\theta_{1,n+1}p_1\omega_1 - \theta_{i+1,n+1}\omega_{n+1}(1-z_{in})\right]
\end{aligned}
$$

Finally, adding the terms from Equations (C.5) and (C.6), the elements of the Jacobian, after explicitly recognizing the conditionality on the error terms $\xi_1$, take the form in Equation (4.13) of Chapter 4.

$$
\begin{aligned}
J_{in} \mid \xi_1 = \frac{1}{R_{i+1}\mid\xi_1}\Bigg\{&\frac{\omega_1}{\omega_{i+1}}\left[(\eta_1\mid\xi_1)(p_1^2 L_1 + p_{i+1}L_{i+1}z_{ih}) + \theta_{1,n+1}p_{n+1}\omega_{n+1}\right] \\
&+ p_{n+1}\left[\theta_{1,n+1}p_1\omega_1 - \theta_{i+1,n+1}\omega_{n+1}(1-z_{in})\right]\Bigg\} \qquad \text{(C.7)}
\end{aligned}
$$

## Appendix D: Computation of the determinant of the Jacobian for the deterministic utility-random maximization (DU-RM) non-additively separable utility model

The elements of the Jacobian are given by:

$$J_{in} = \frac{\partial \varepsilon_{i+1}}{\partial x_{n+1}}, \quad i, n = 1,2,...,M-1, \tag{D.1}$$

where the error term of alternative $i+1$ ($i = 1,2,...,M-1.$) is:

$$\varepsilon_{i+1} = V_1 - V_{i+1} + \varepsilon_1 \tag{D.2}$$

Then, the $in^{th}$ element of the Jacobian is:

$$J_{in} = \frac{\partial (V_1 - V_{i+1} + \varepsilon_1)}{\partial x_{n+1}} = \frac{\partial V_1}{\partial x_{n+1}} - \frac{\partial V_{i+1}}{\partial x_{n+1}}. \tag{D.3}$$

Given that $x_1 = E - \sum_{r=2}^{K} p_r x_r$, the first term of $J_{in}$ is given by:

$$\begin{aligned}
\frac{\partial V_1}{\partial x_{n+1}} &= \frac{\partial}{\partial x_{n+1}}\left[\ln(\tilde{\pi}_1) + (\alpha_1 - 1)\ln\left(\frac{x_1}{\gamma_1} + 1\right) - \ln p_1\right] \\
&= \frac{1}{\tilde{\pi}_1}\frac{\partial \tilde{\pi}_1}{\partial x_{n+1}} + (\alpha_1 - 1)\frac{\partial}{\partial x_{n+1}}\ln\left(\frac{x_1}{\gamma_1} + 1\right) \\
&= \frac{1}{\tilde{\pi}_1}\theta_{1,n+1}\left(\frac{x_{n+1}}{\gamma_{n+1}} + 1\right)^{\alpha_{n+1}-1} - \frac{(\alpha_1 - 1)p_{n+1}}{\gamma_1}\left(\frac{x_1}{\gamma_1} + 1\right)^{-1} \\
&= \frac{1}{\tilde{\pi}_1}\theta_{1,n+1} p_{n+1}\omega_{n+1} + L_1 p_1 p_{n+1}
\end{aligned} \tag{D.4}$$

The second term of the right-hand side of Equation (D.3) is

$$\begin{aligned}
-\frac{\partial V_{i+1}}{\partial x_{n+1}} &= -\frac{\partial}{\partial x_{n+1}}\left[\ln(\tilde{\pi}_{i+1}) + (\alpha_{i+1} - 1)\ln\left(\frac{x_{i+1}}{\gamma_{i+1}} + 1\right) - \ln p_{i+1}\right] \\
&= -\frac{1}{\tilde{\pi}_{i+1}}\frac{\partial \tilde{\pi}_{i+1}}{\partial x_{n+1}} - (\alpha_{i+1} - 1)\frac{\partial}{\partial x_{n+1}}\ln\left(\frac{x_{i+1}}{\gamma_{i+1}} + 1\right) z_{in}
\end{aligned} \tag{D.5}$$

Now, using the fact that Given that $x_1 = E - \sum_{r=2}^{K} p_r x_r$,

$$
\begin{aligned}
-\frac{\partial V_{i+1}}{\partial x_{n+1}} &= -\frac{1}{\tilde{\pi}_{i+1}}\left\{\frac{\partial}{\partial x_{n+1}}\left\{\sum_{\substack{m\neq i\\ m\neq 1}}\theta_{i+1,m}\frac{\gamma_m}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right]\right\}(1-z_{in})\right.\\
&\quad\left.+\frac{\partial}{\partial x_{n+1}}\left\{\theta_{1,i+1}\frac{\gamma_1}{\alpha_1}\left[\left(\frac{x_1}{\gamma_1}+1\right)^{\alpha_1}-1\right]\right\}-(\alpha_{i+1}-1)\frac{\partial}{\partial x_{n+1}}\ln\left(\frac{x_{i+1}}{\gamma_{i+1}}+1\right)z_{in}\right. \\
&= -\frac{1}{\tilde{\pi}_{i+1}}\left\{\theta_{i+1,n+1}\left(\frac{x_{n+1}}{\gamma_{n+1}}+1\right)^{\alpha_{n+1}-1}(1-z_{in})+\theta_{1,i+1}\left(\frac{x_1}{\gamma_1}+1\right)^{\alpha_1-1}(-p_1)\right\}+L_{n+1}p_{n+1}z_{in}\\
&= -\frac{1}{\tilde{\pi}_{i+1}}\left\{\theta_{i+1,n+1}p_{n+1}\omega_{n+1}(1-z_{in})+\theta_{1,i+1}p_1^2\omega_1\right\}+L_{n+1}p_{n+1}z_{in}
\end{aligned}
\tag{D.6}
$$

Finally, adding the terms from Equations (D.4) and (D.6), the elements of the Jacobian take the form in Equation (4.19) of Chapter 4.

$$
J_{in} = p_{n+1}\omega_{n+1}\left[\frac{\theta_{1,n+1}}{\tilde{\pi}_1}-\frac{\theta_{i+1,n+1}}{\tilde{\pi}_{i+1}}(1-z_{in})\right]+p_1^2\omega_1\frac{\theta_{1,i+1}}{\tilde{\pi}_{i+1}}+p_{n+1}\left[p_1L_1+L_{n+1}z_{in}\right] \tag{D.7}
$$

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